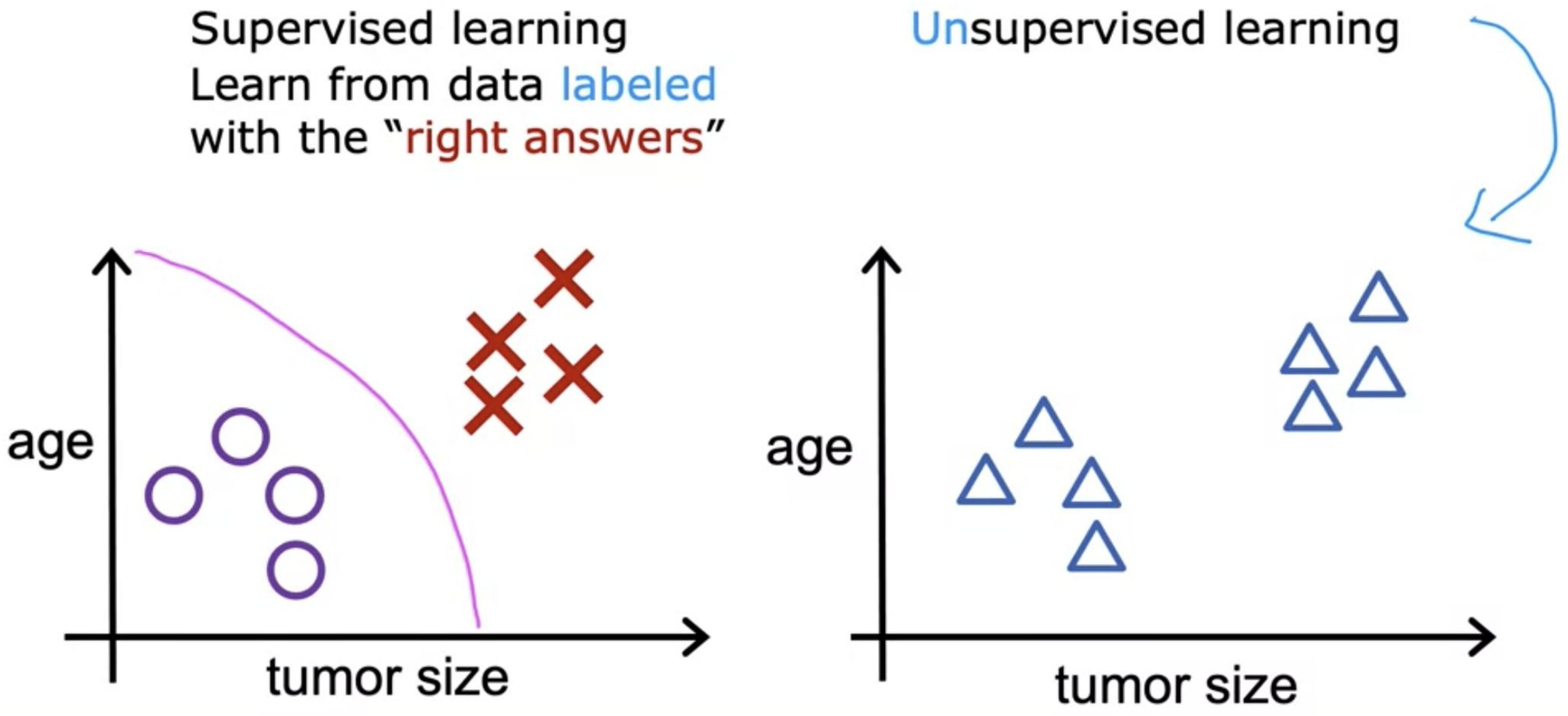
AIBridge

Lecture 8

Introducing Unsupervised Learning

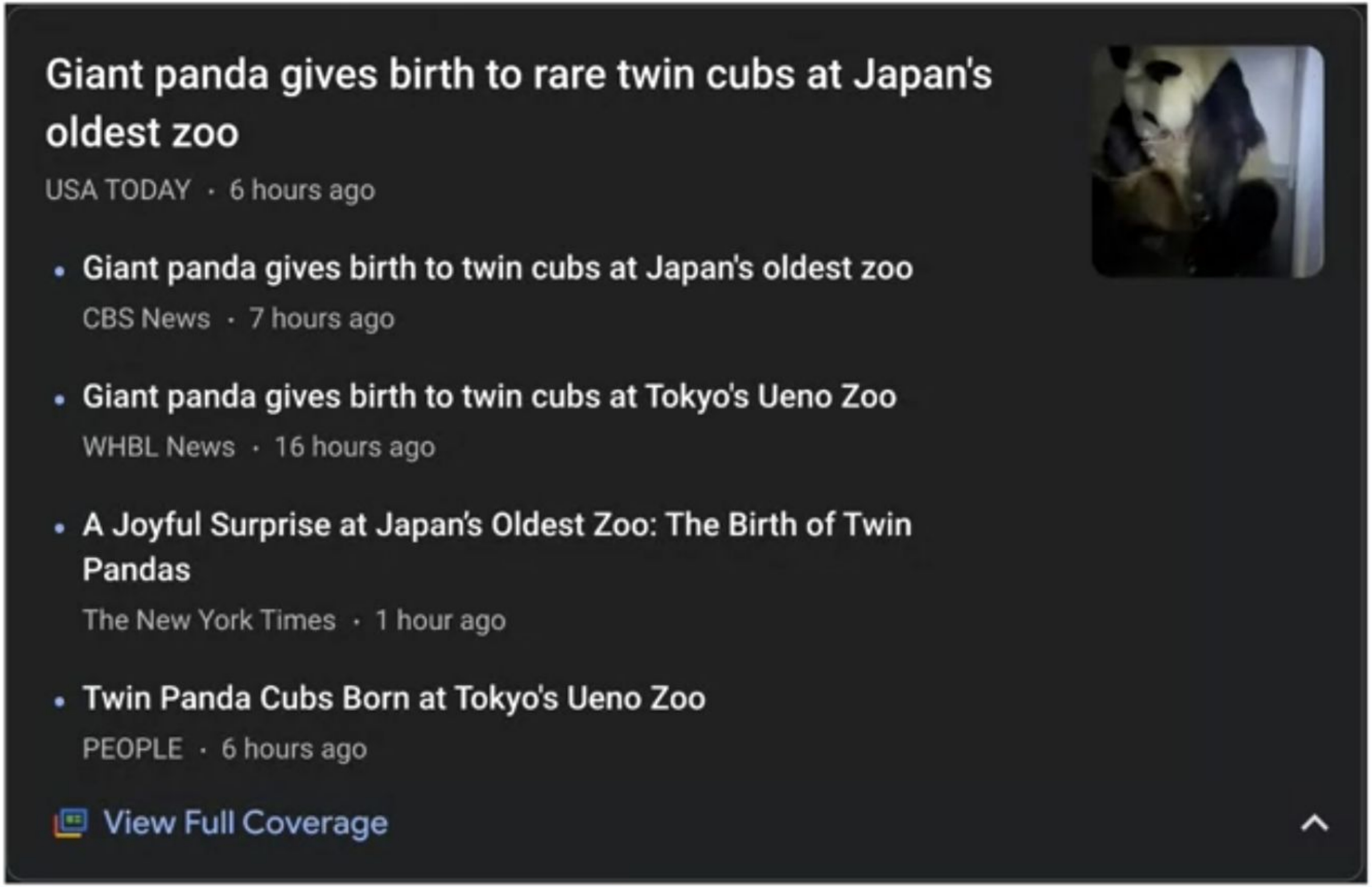


Credit: Andrew Ng, [Machine Learning]

Unsupervised Learning

Clustering
Dimension reduction

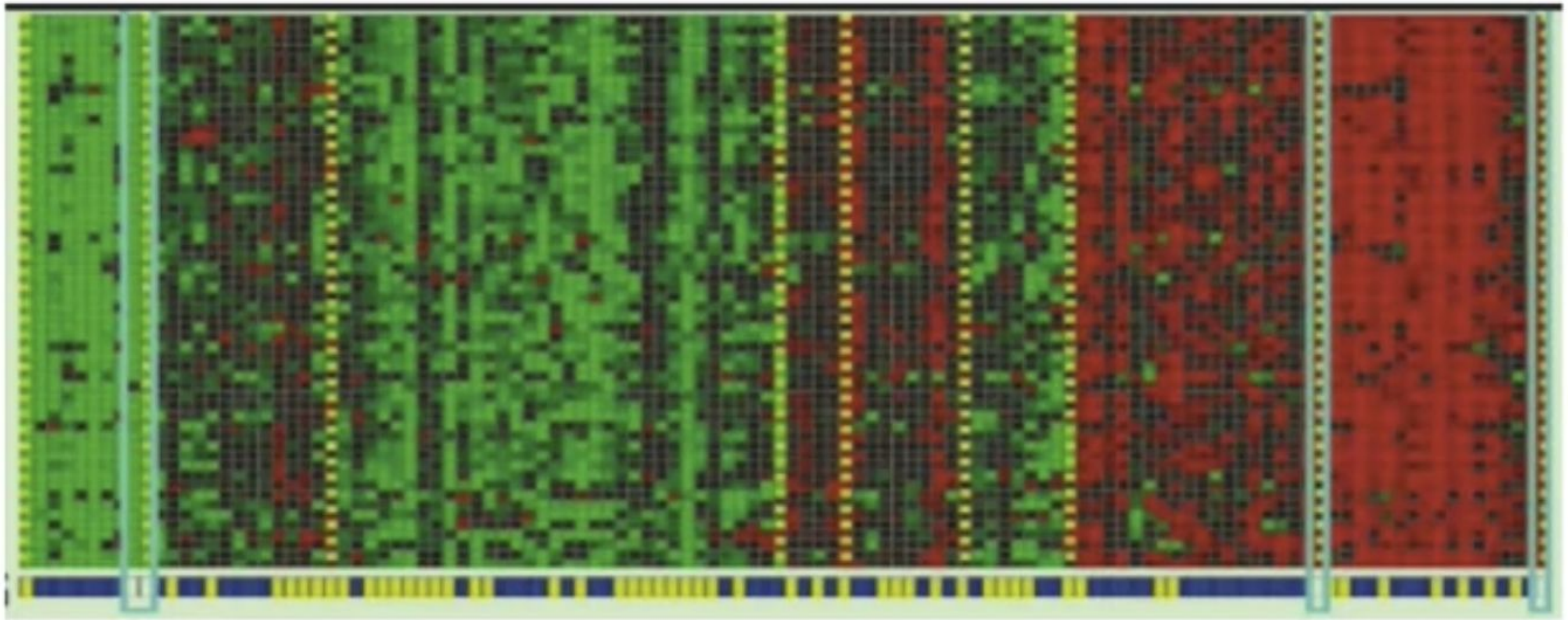
Clustering: Google news



Credit: Andrew Ng, [Machine Learning]

Clustering: DNA microarray

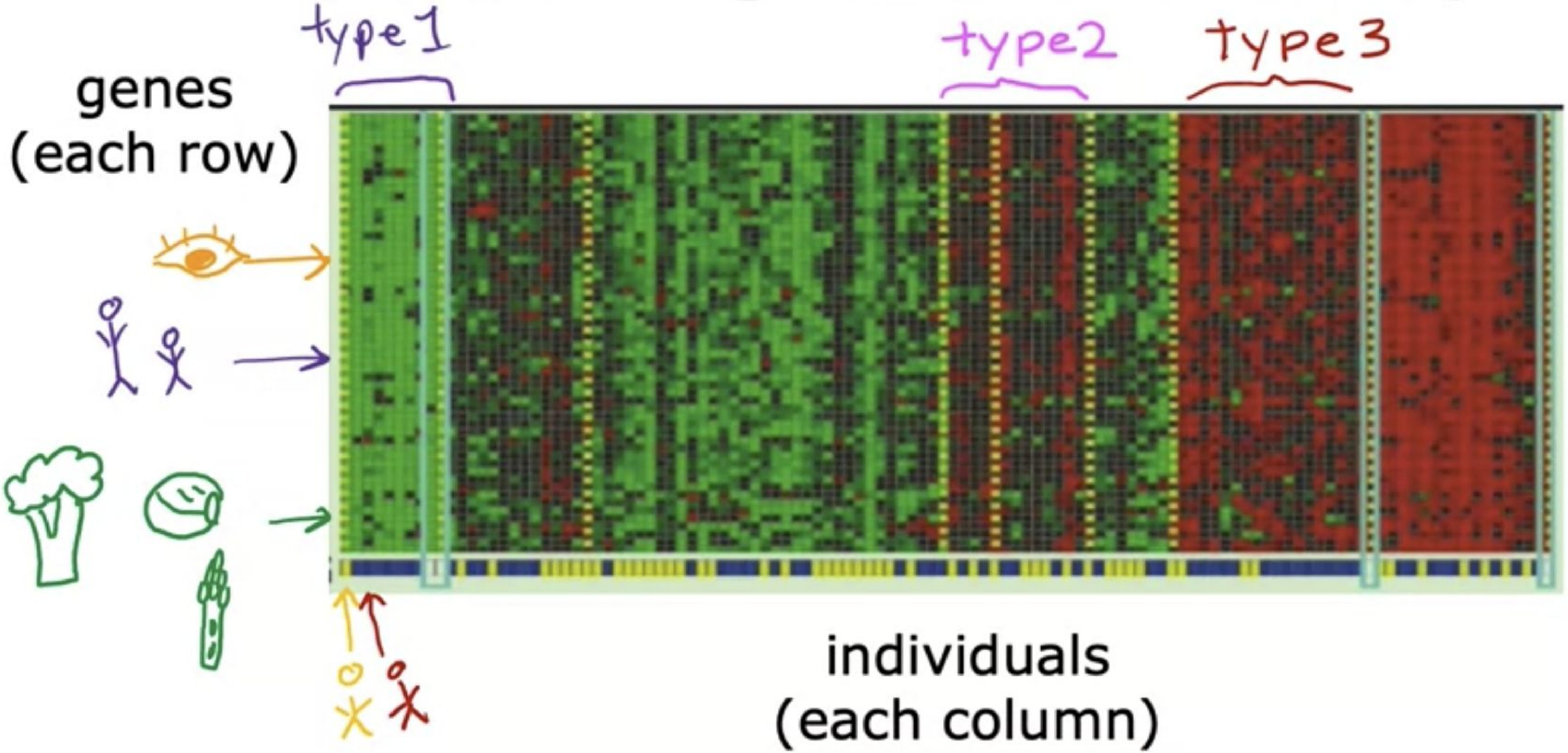
genes
(each row)



individuals
(each column)

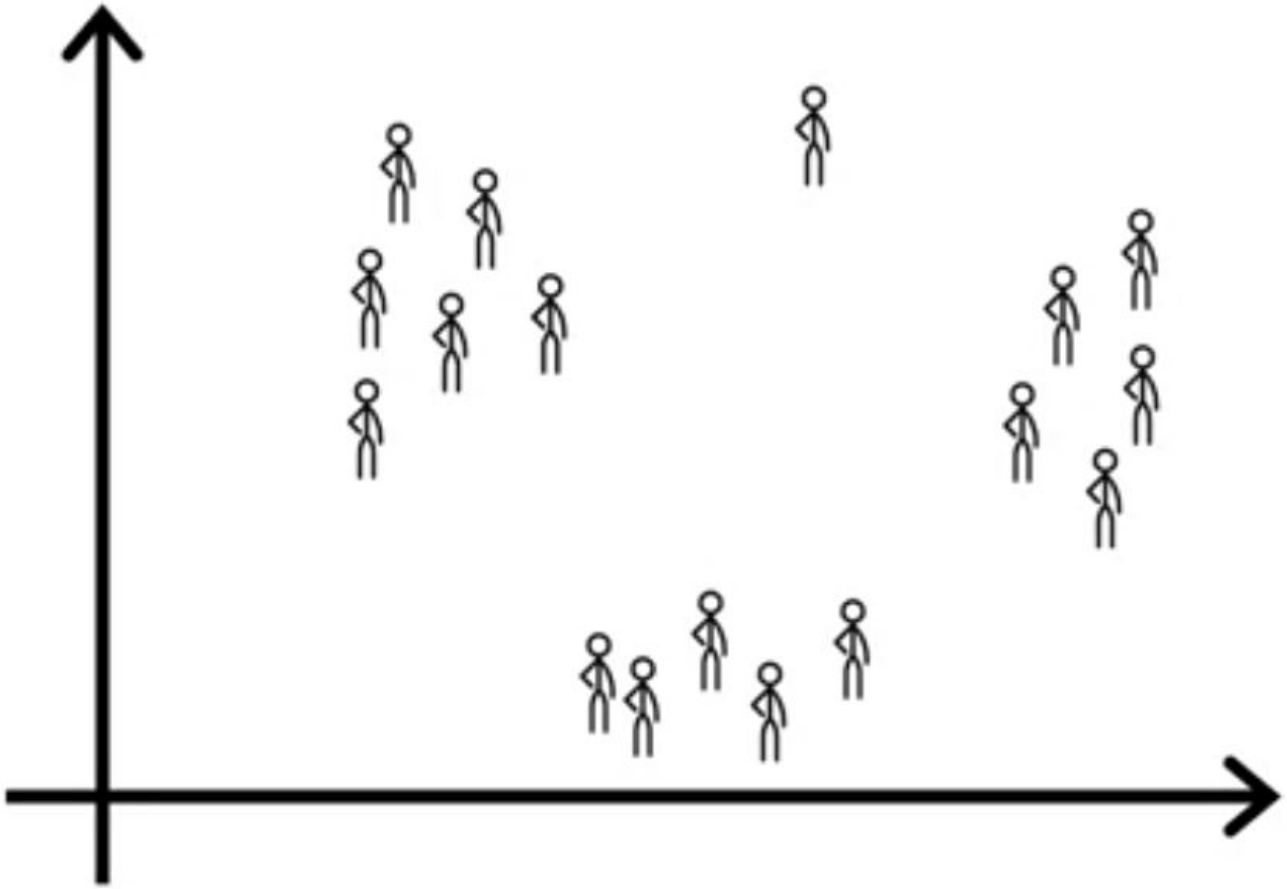
Credit: Andrew Ng, [Machine Learning]

Clustering: DNA microarray



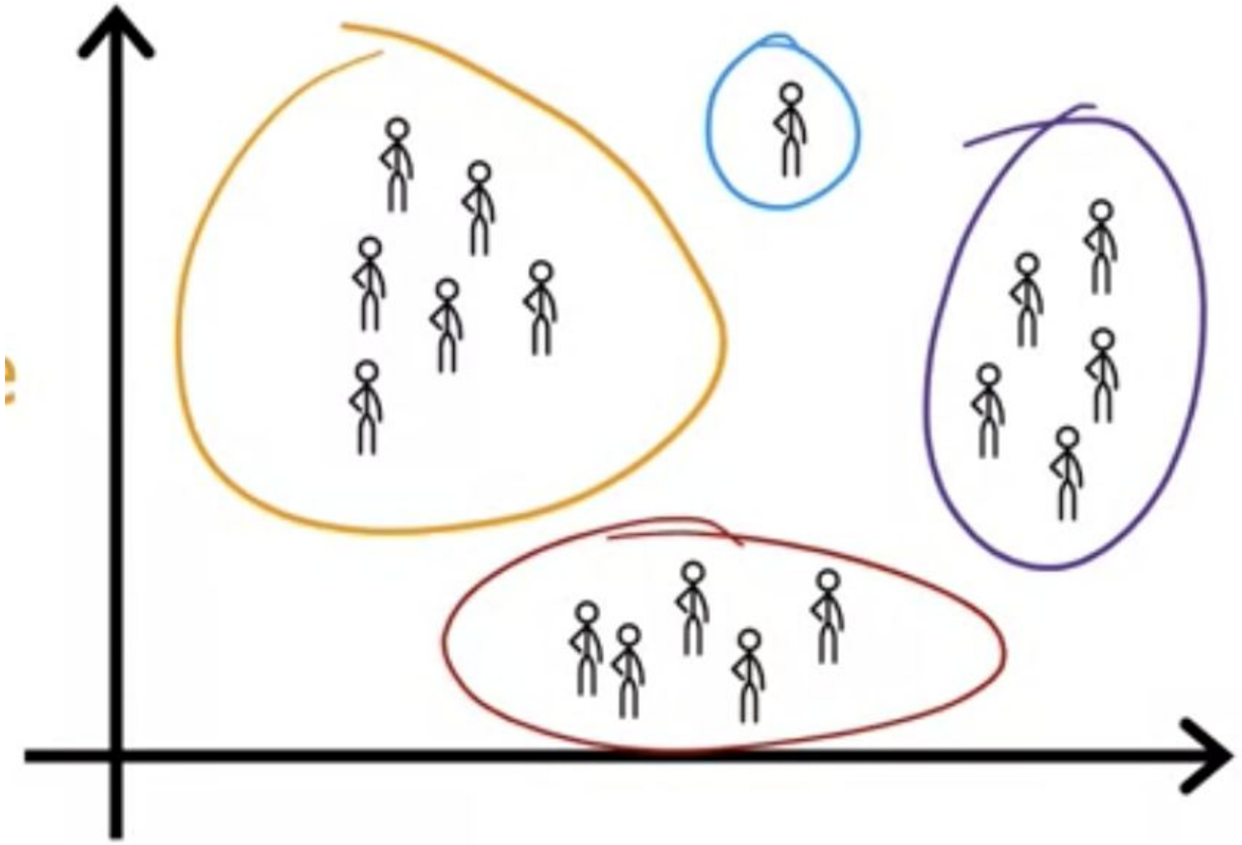
Credit: Andrew Ng, [Machine Learning]

Clustering: Grouping customers



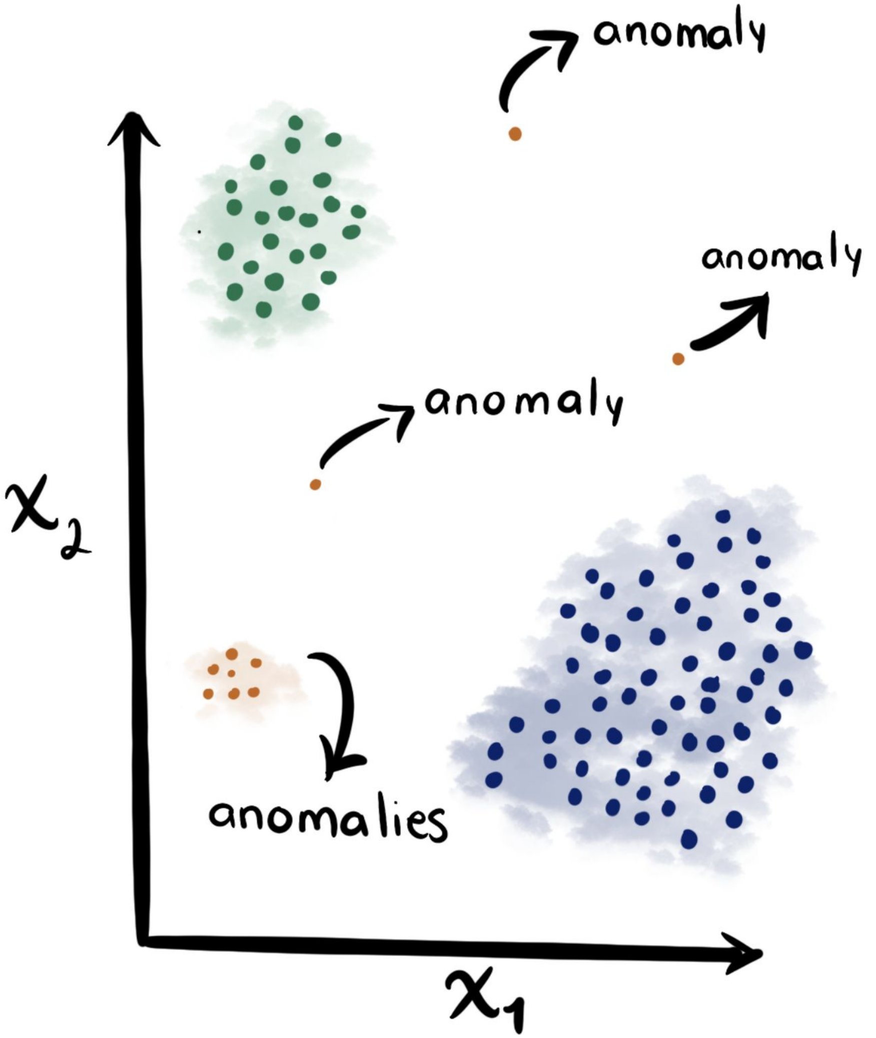
Credit: Andrew Ng, Machine Learning

Grouping Customers



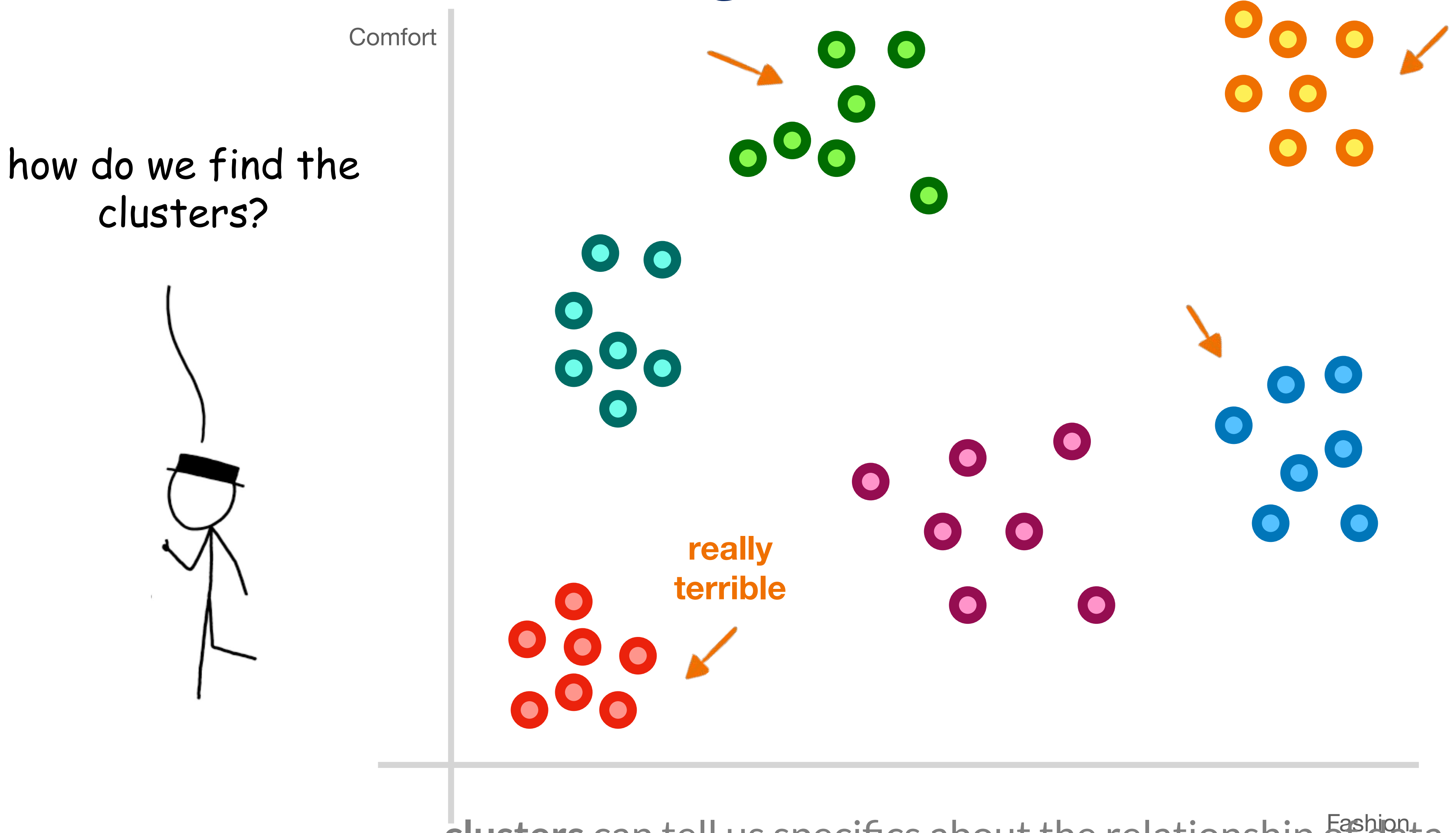
Credit: Andrew Ng, [Machine Learning]

Anomaly Detection



Credit: [Anomaly Detection]

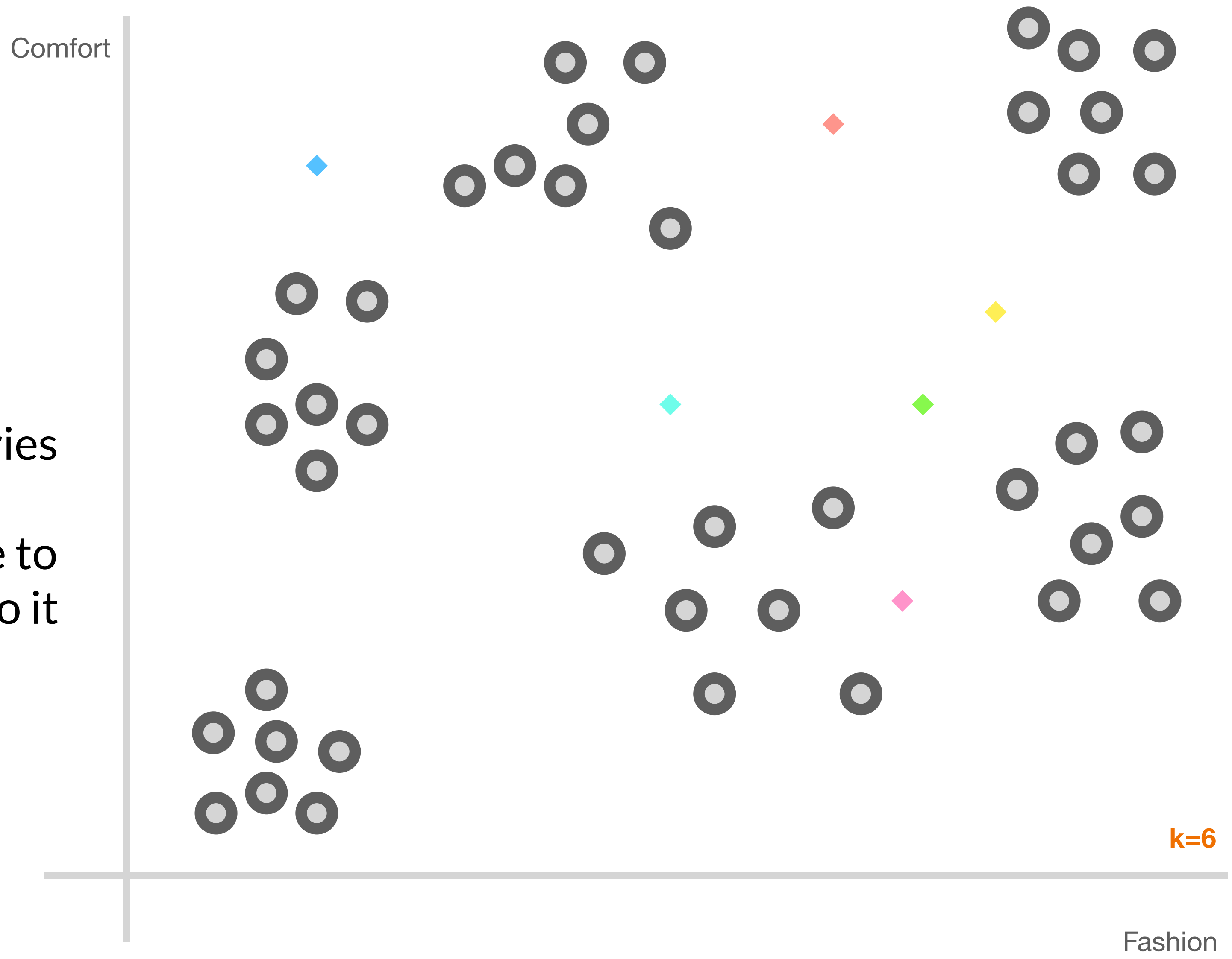
K-means clustering



clusters can tell us specifics about the relationship of data

...even if they are unlabeled! → **unsupervised learning!**

1. pick a K-number of clusters
2. randomly pick a series of “**centroids**”
3. assign each particle to the **centroid** closest to it



1. pick a K-number of clusters
2. randomly pick a series of "**centroids**"
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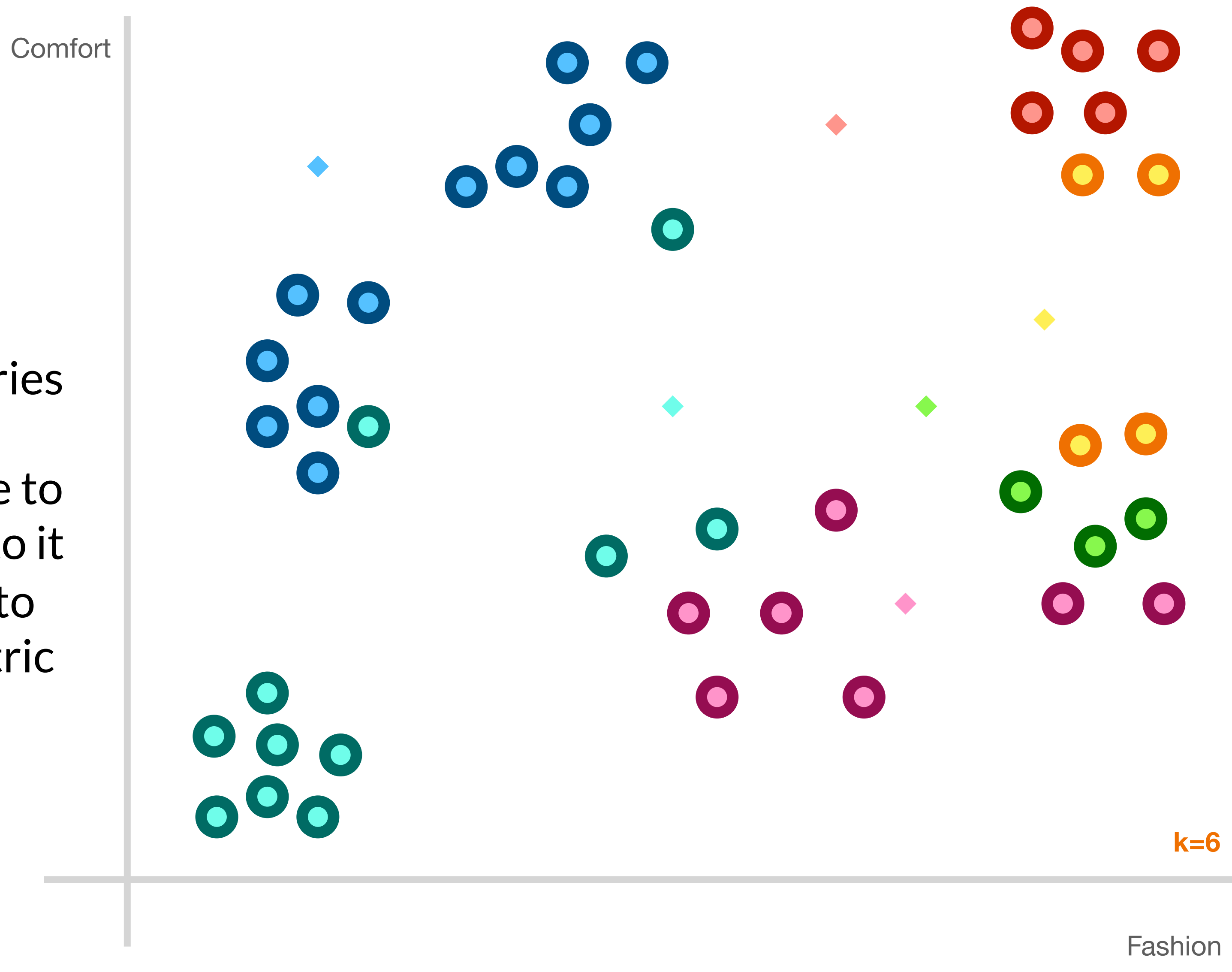
4. move the **centroid** to the weighted geometric center of samples assigned to it



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2. randomly pick a series of “**centroids**”
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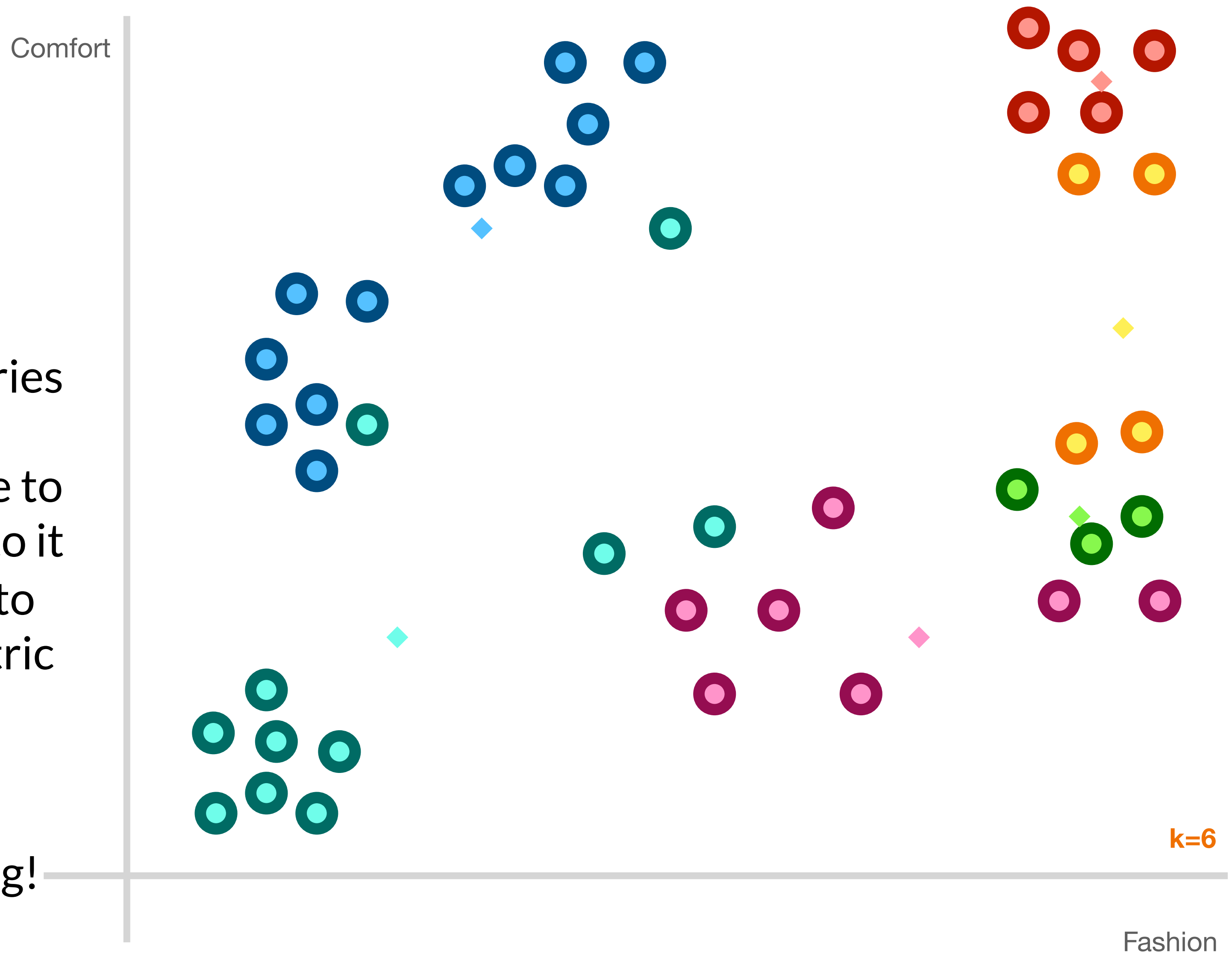
4. move the **centroid** to the weighted geometric center of samples assigned to it
5. Repeat 3-4 until centroids stop moving!



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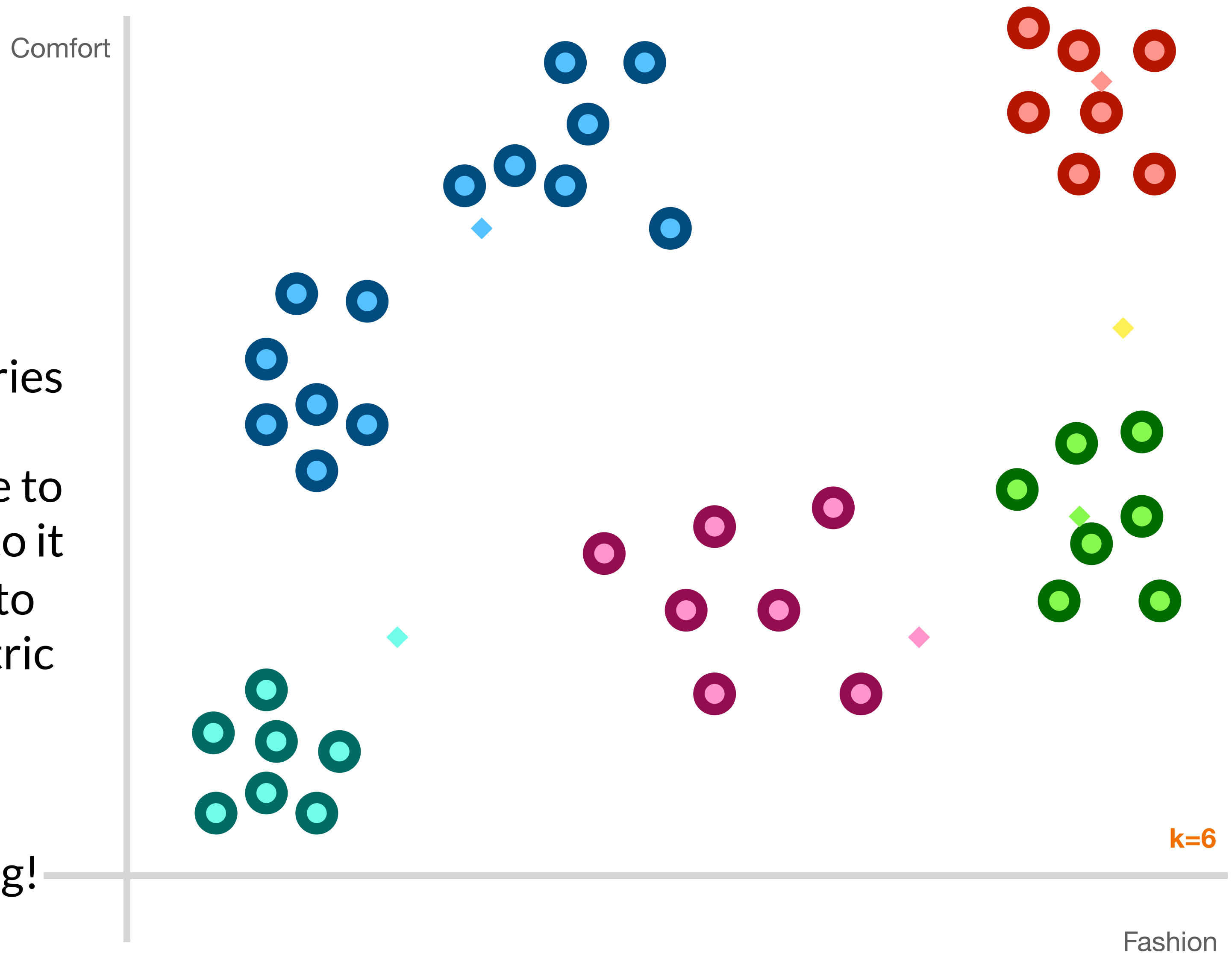

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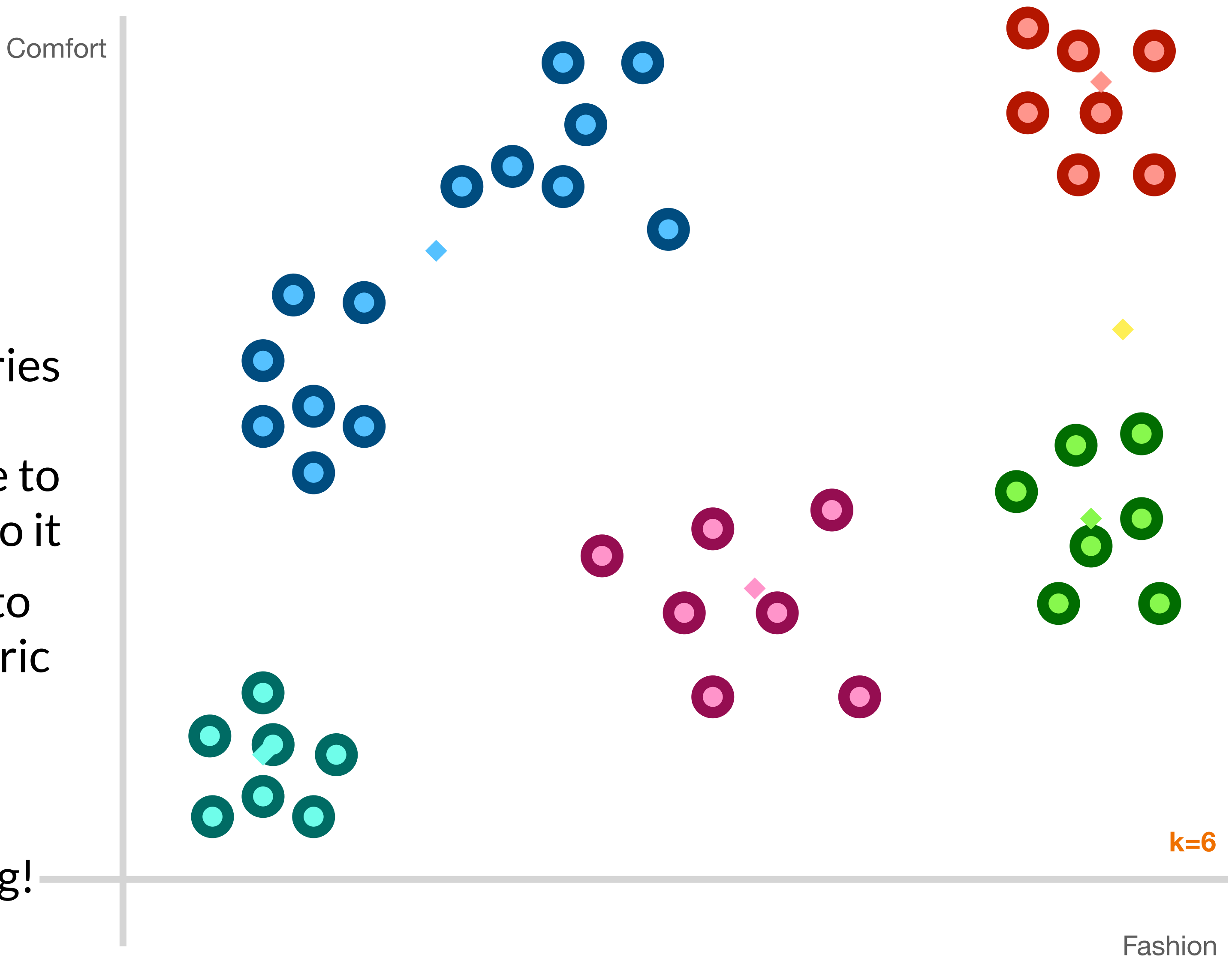


Did we get back the same clusters?

Nope. And that’s OK.

Did we get back the same clusters?

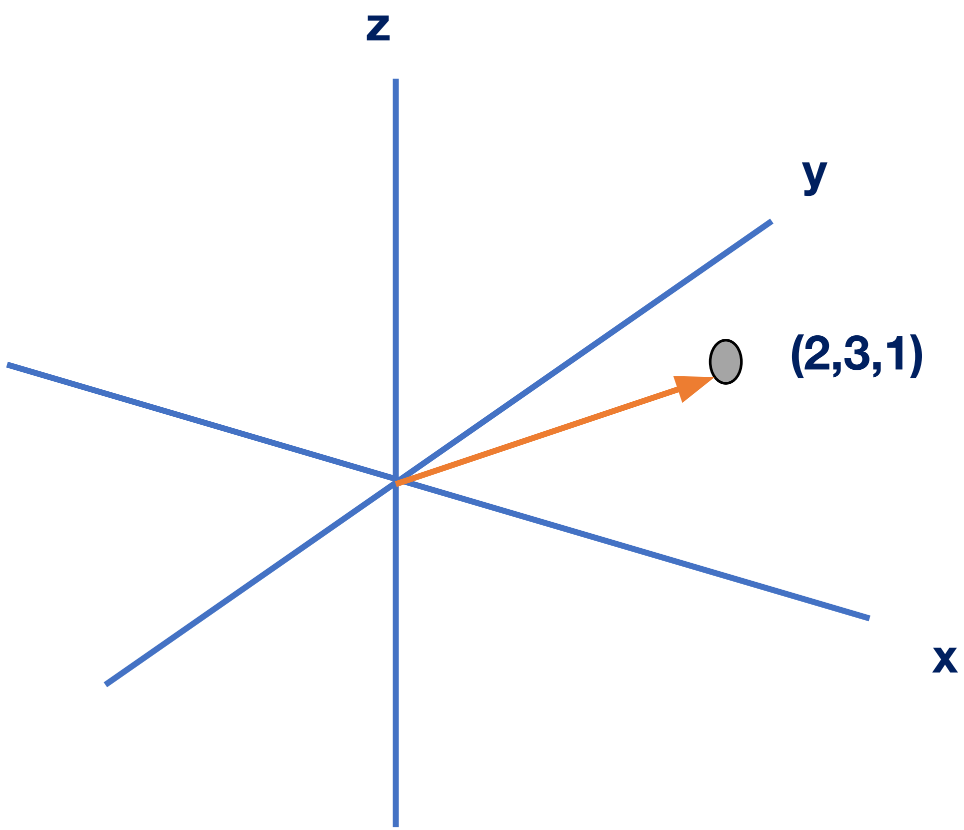
Nope. And that's OK.

K-means is an *indeterministic* algorithm—it has built-in randomness

Unsupervised Learning

Clustering
Dimension reduction

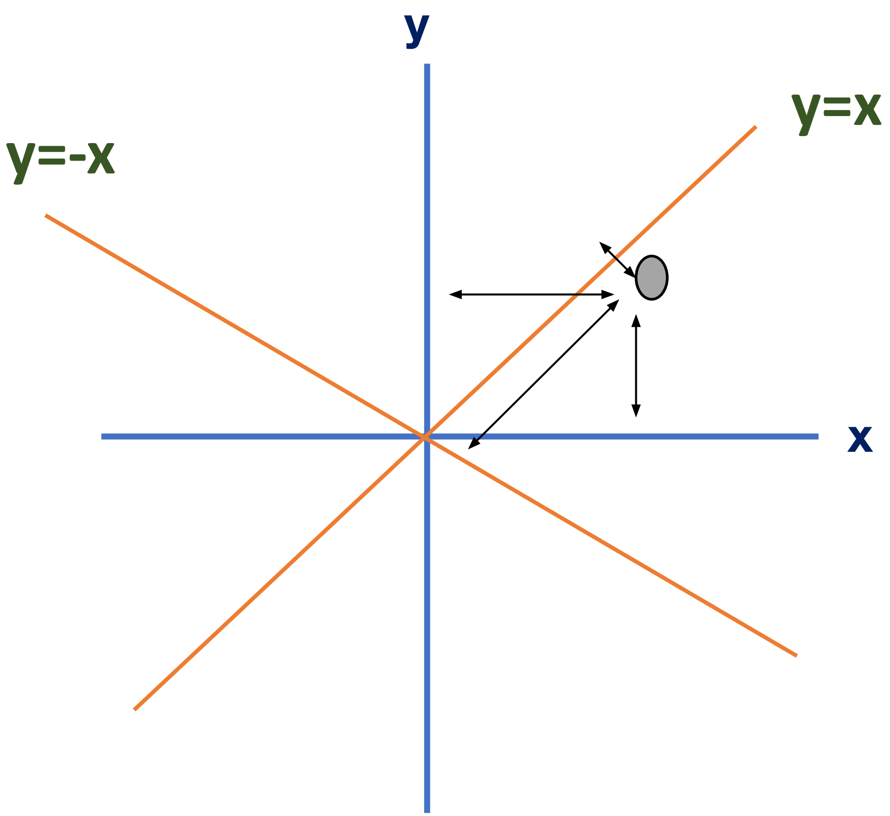
Exploring Dimensions and Basis Vectors



(2,3,1) is a datapoint.

$\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ is the vector to said datapoint.

Exploring Dimensions and Basis Vectors



This gray point can be expressed as 3 blocks from x axis and 2 blocks from the y axis.

It can also be expressed as 1 block from y = -x and 3 blocks from y=x

Motivation for Dimension Reduction

Complex systems often must be modeled with large datasets, having dozens of columns.

Often, several columns can be adding similar information to the model. So, there is a certain level of *redundancy.*

Additionally, datasets with too many features may be difficult to represent graphically.

Individual	Height (cm)	Weight (kg)	Income ($)	Number of Children
Person A	165	65	60,000	2
Person B	168	63	100,000	5
Person C	159	82	50,000	1
Person D	183	68	90,000	4
Person E	187	87	110,000	5
Person F	189	89	95,000	4

Four dimensions; can't even be graphed!

Motivation for Dimension Reduction

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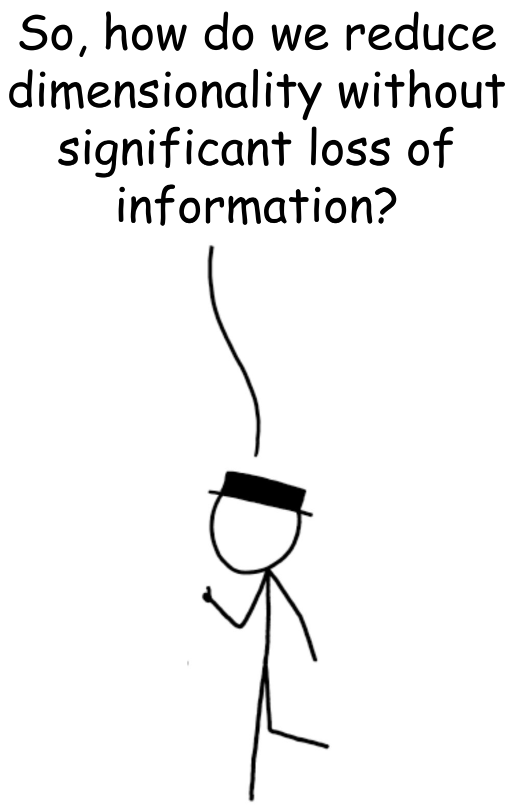
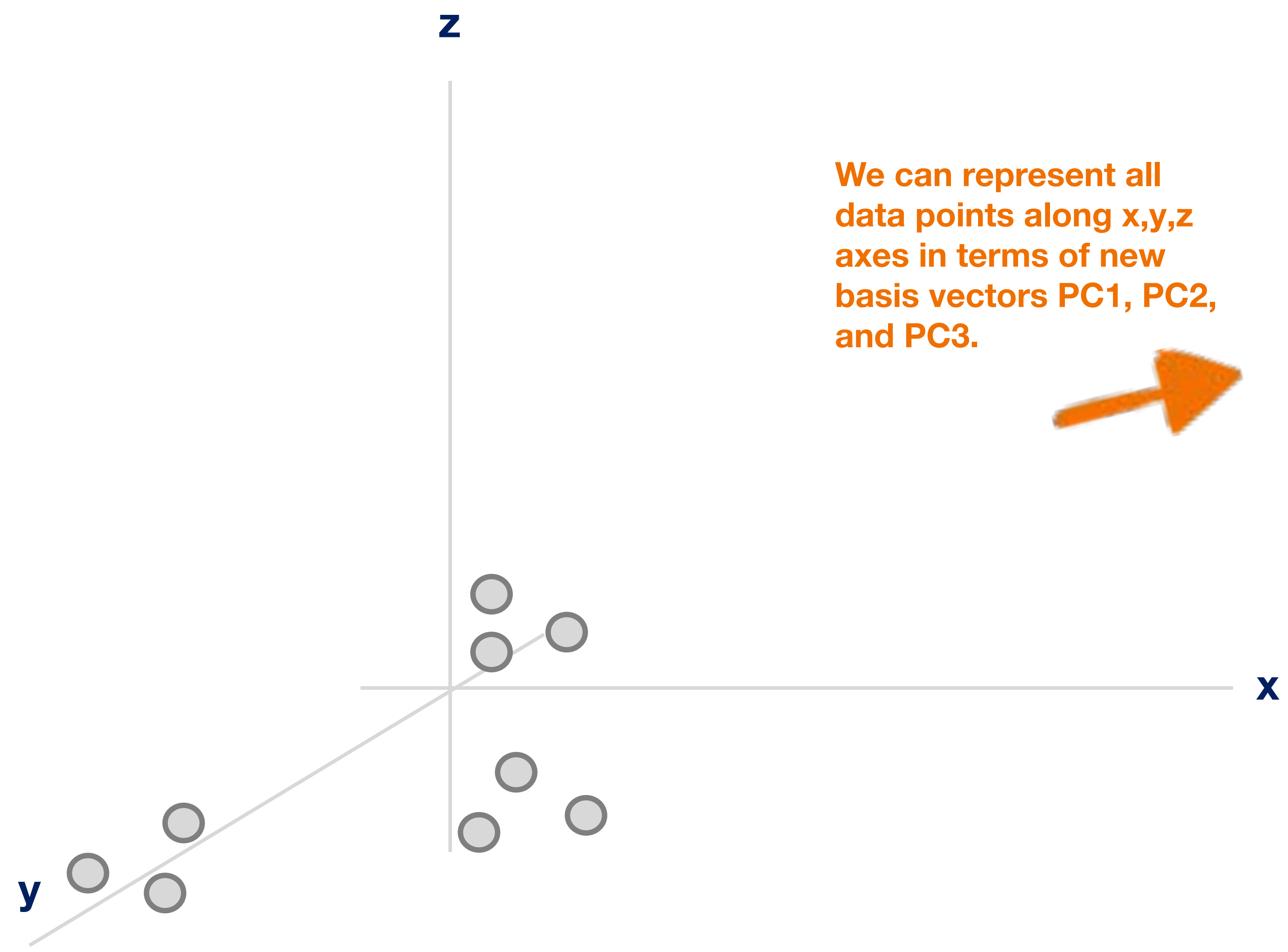


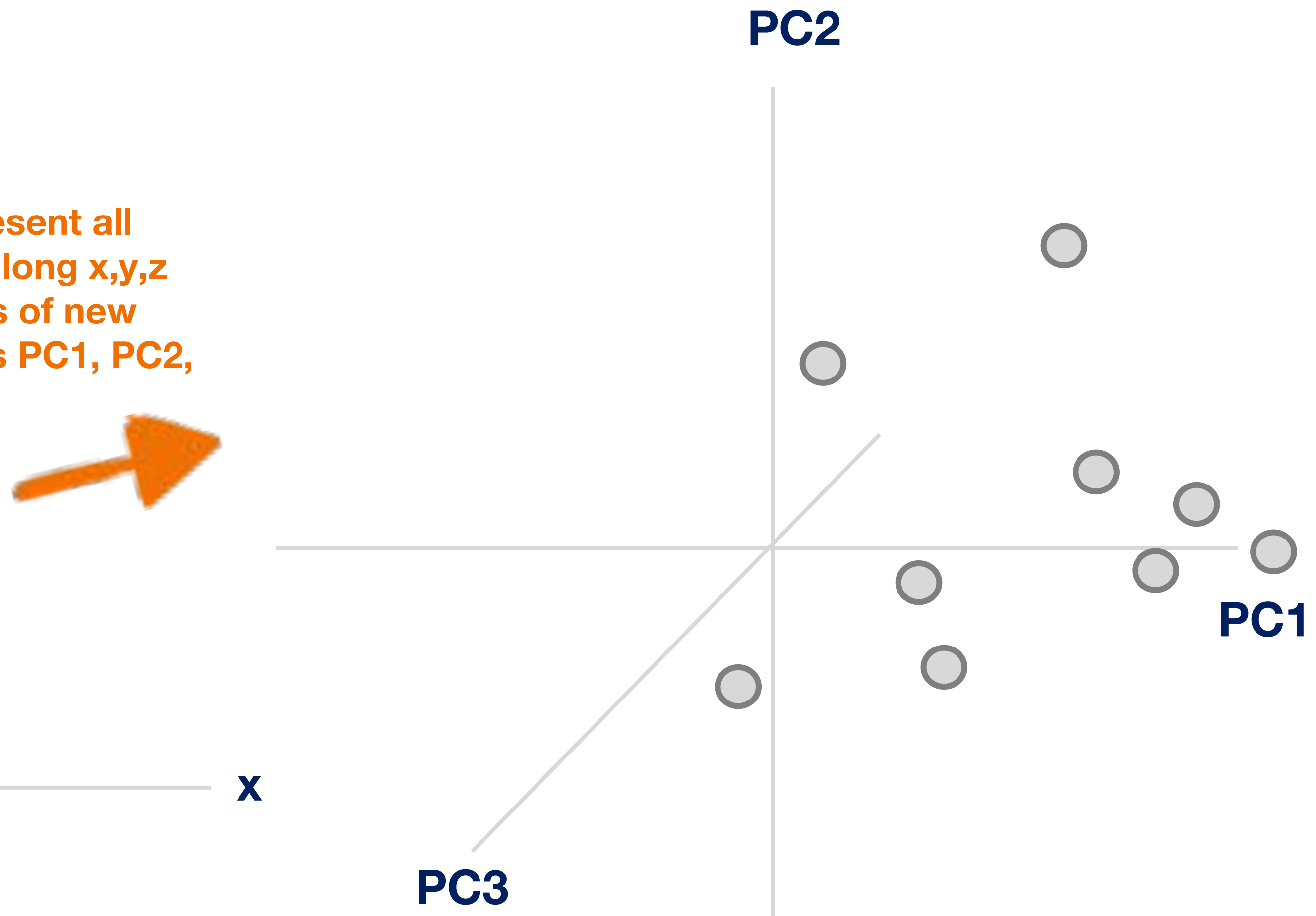
Enter…

Principal Component Analysis

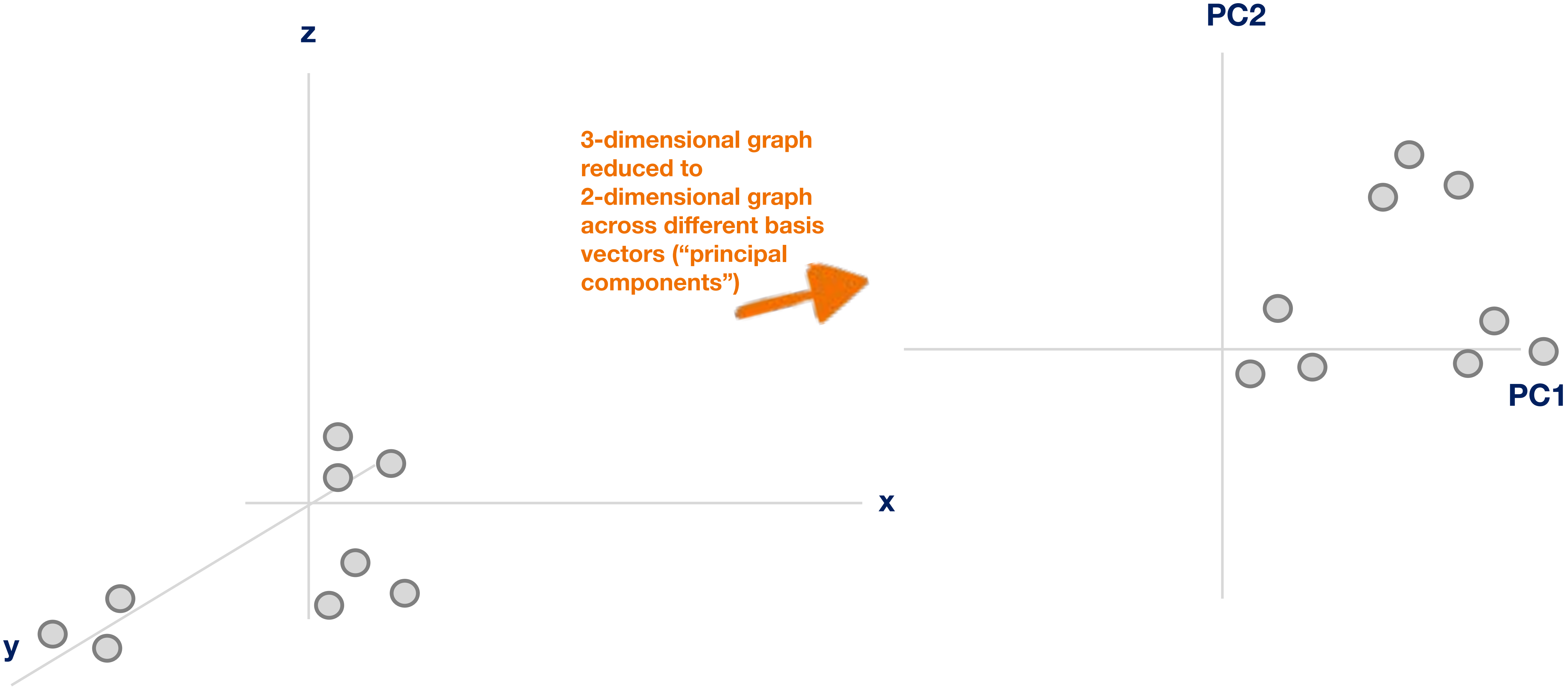
Principal Component Analysis



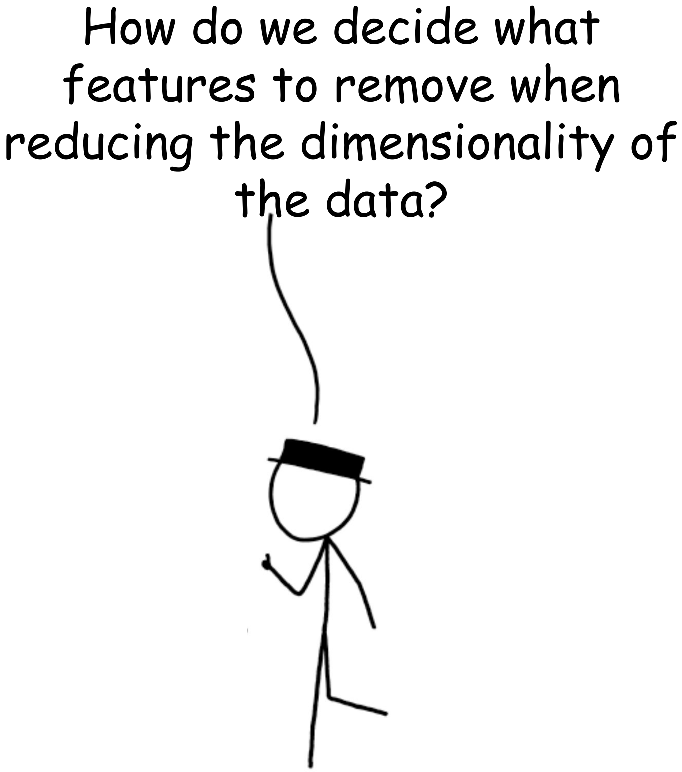
We can represent all data points along x,y,z axes in terms of new basis vectors PC1, PC2, and PC3.



Principal Component Analysis



Principal Component Analysis



Principal Components

Think of these as new axes that we are orienting our data across.

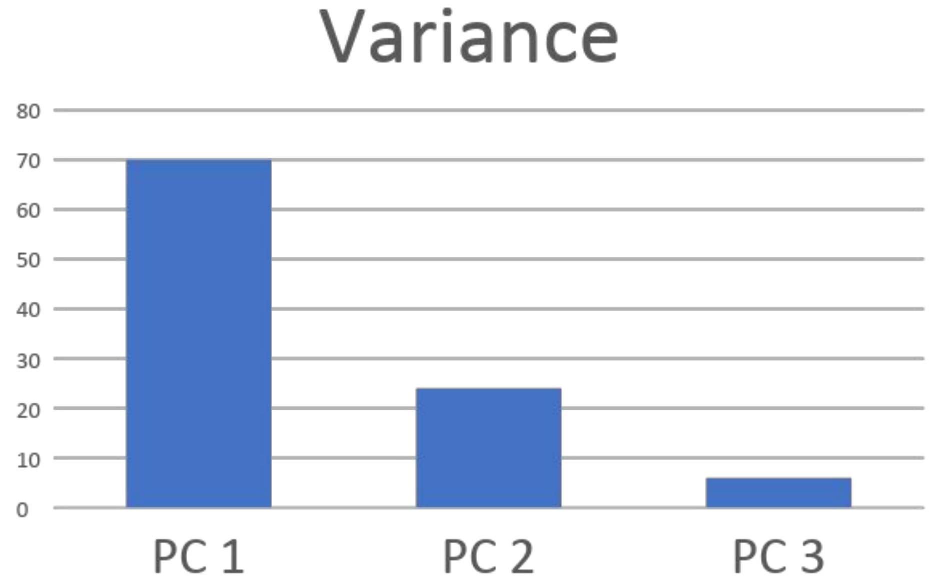
So instead of x,y, z, rather some linear combination of them.

They are done such that each principal component is uncorrelated with the others, so that translation across each component indicates different information. **So, they represent directions of maximal variance.**

This allows differences between data points to become more prominent

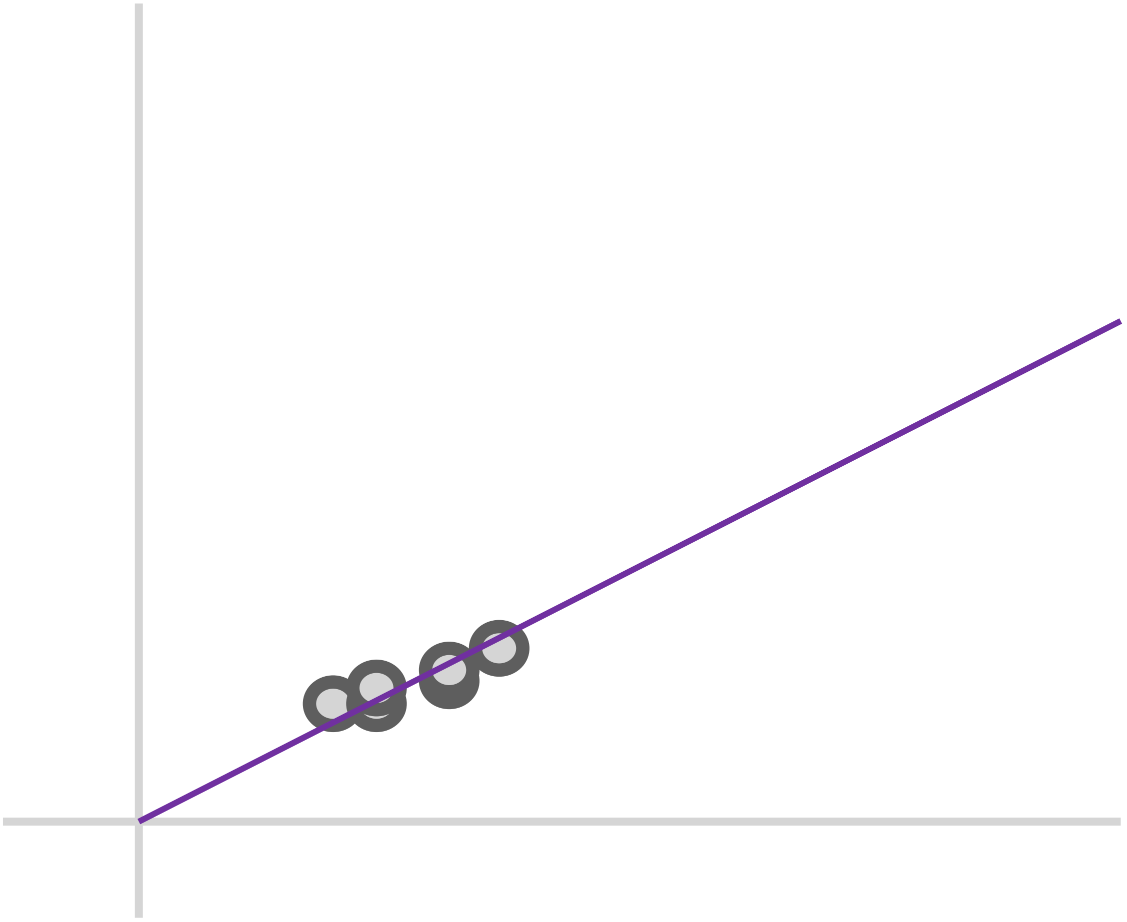
How do we decide what features to remove when reducing the dimensionality of the data?

Represents percentage of variance for each PC. Notice how PC1 has the most and it drops after that.



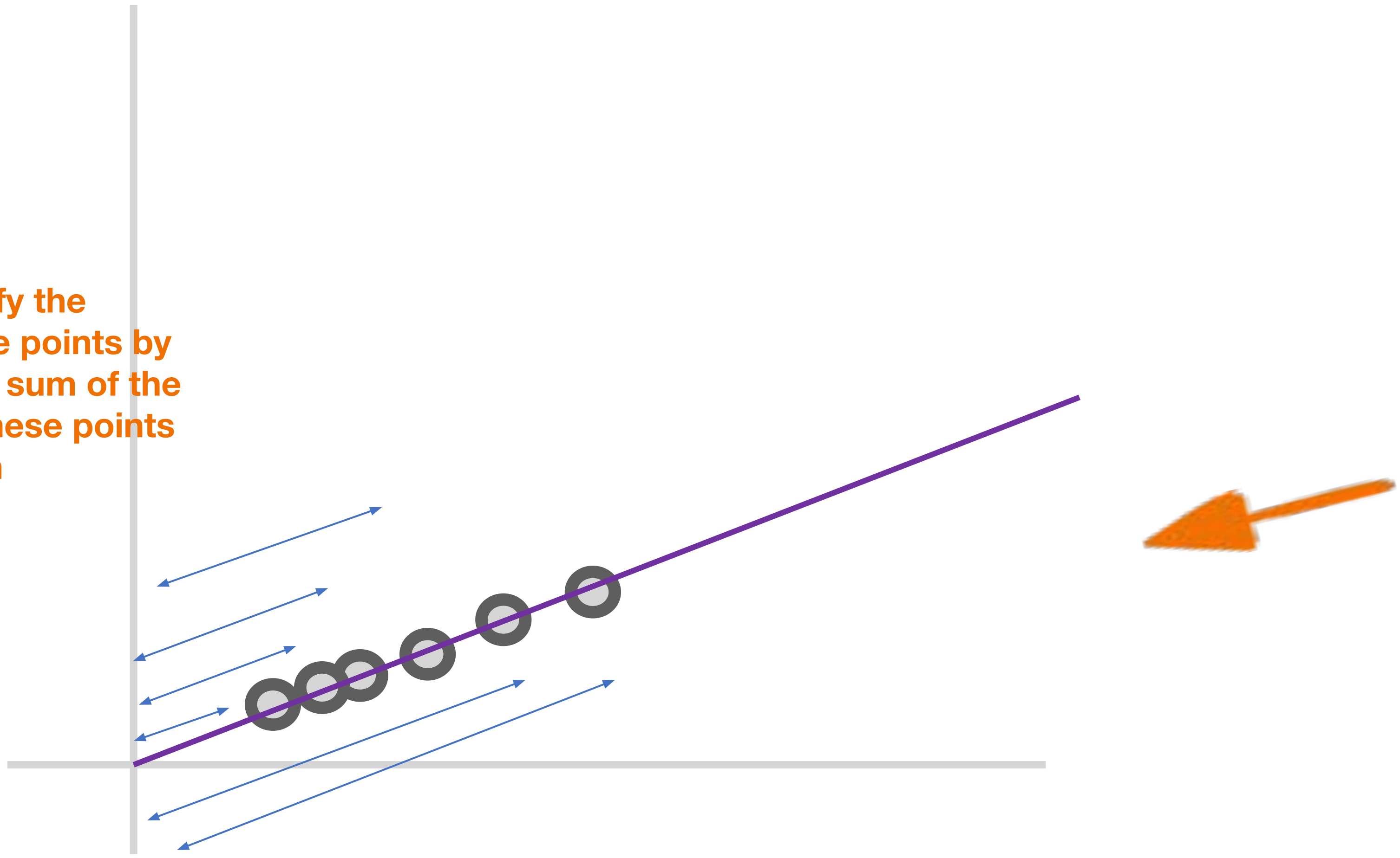
Since PC3 accounts for a very small percentage of overall variance, we can remove it. This is how PCA reduces dimensionality

Principal Component Analysis



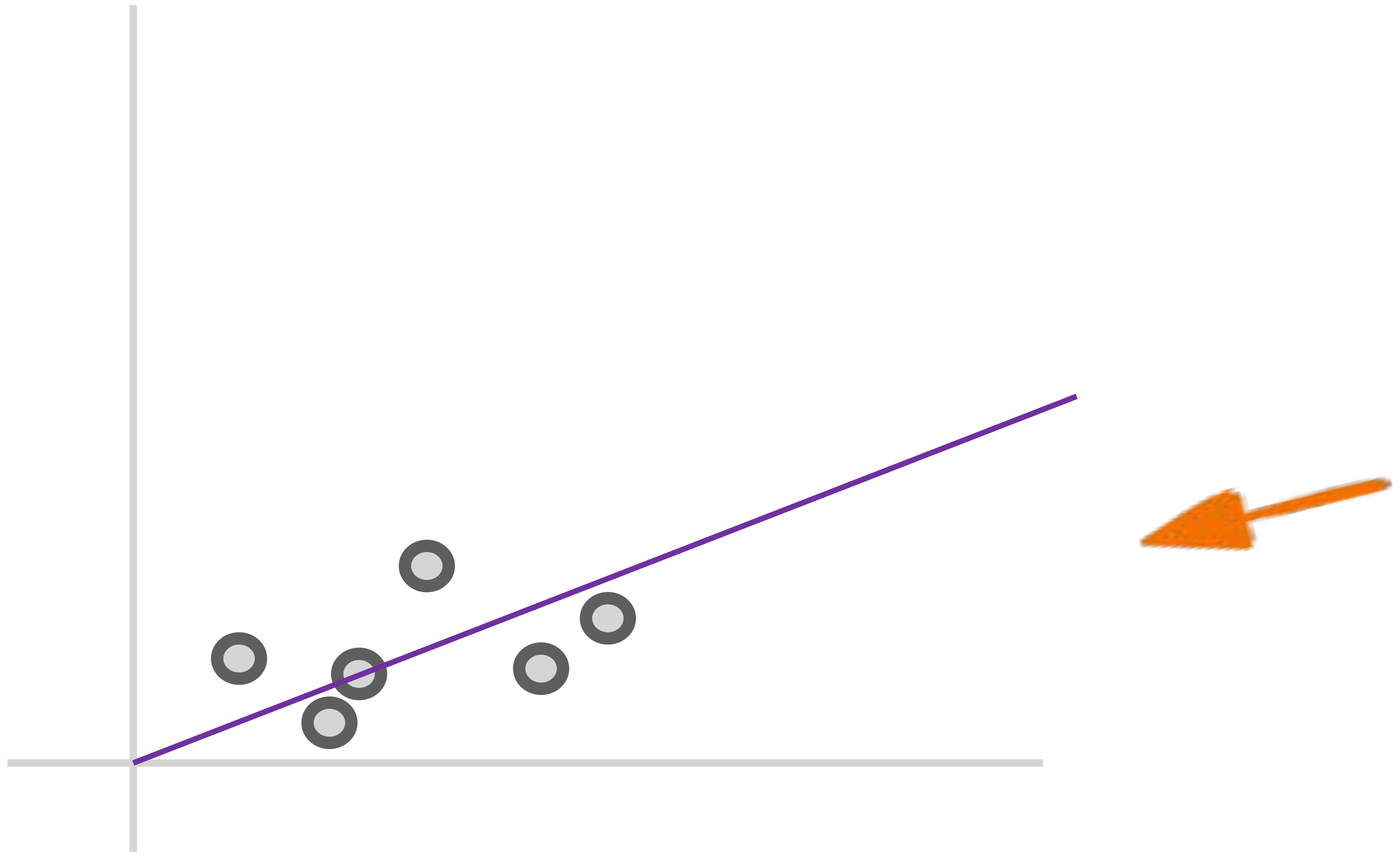
Principal Component Analysis

We can quantify the "spread" of the points by measuring the sum of the distances of these points from the origin



Notice how the points are a lot further away on average from each other and from the origin.

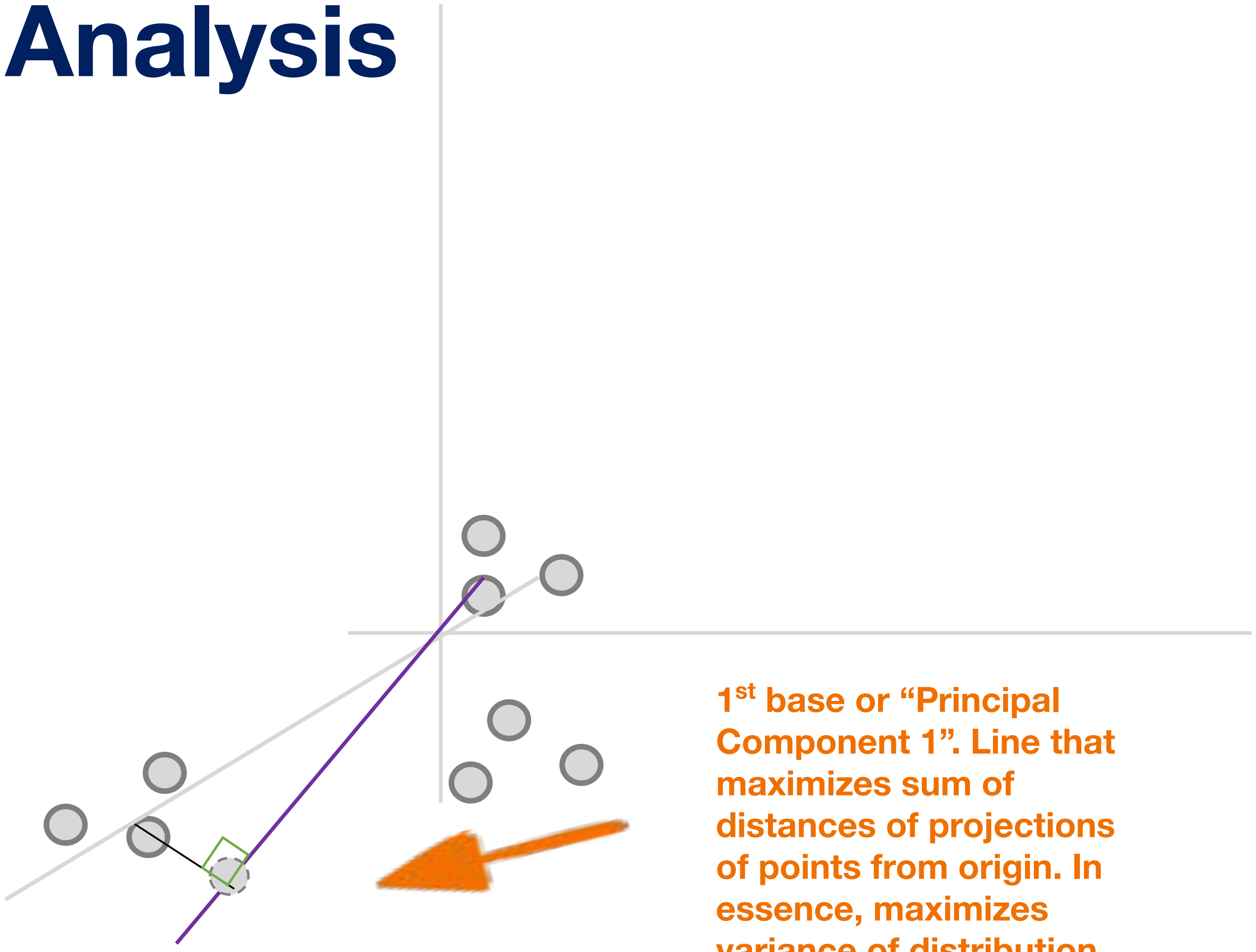
Principal Component Analysis



Not all points lie on the line! How do we quantify the spread or variance of the points?

Principal Component Analysis

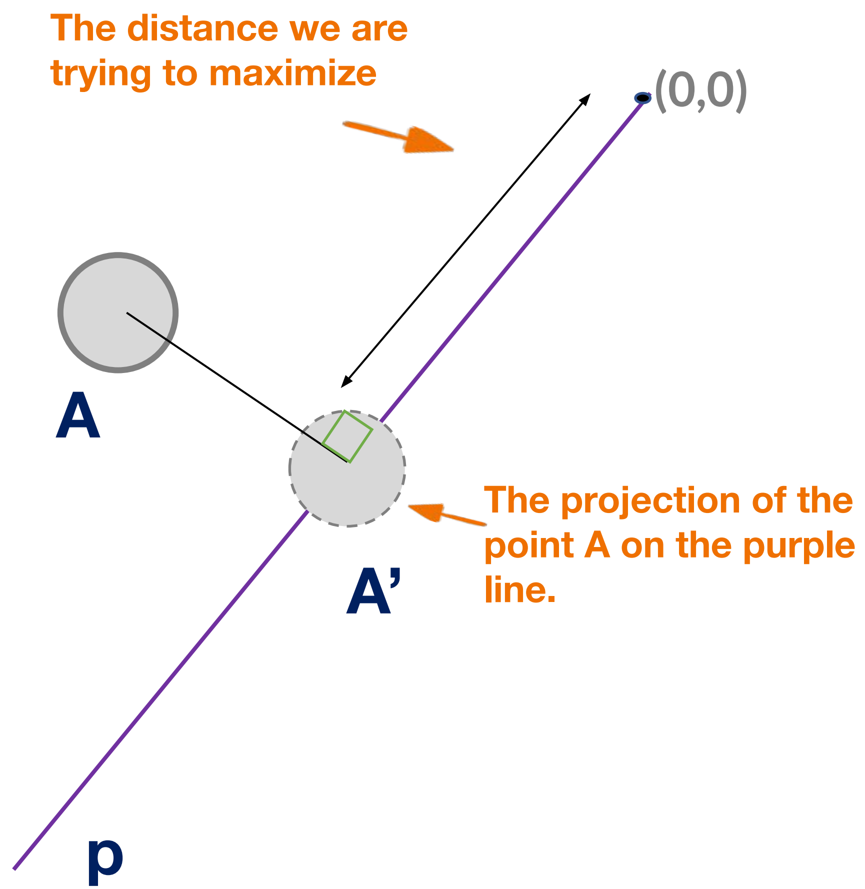
The degree to which a base aligns with the variance represents the amount of information separations along that basis can convey.



1^{st} base or "Principal Component 1". Line that maximizes sum of distances of projections of points from origin. In essence, maximizes variance of distribution.

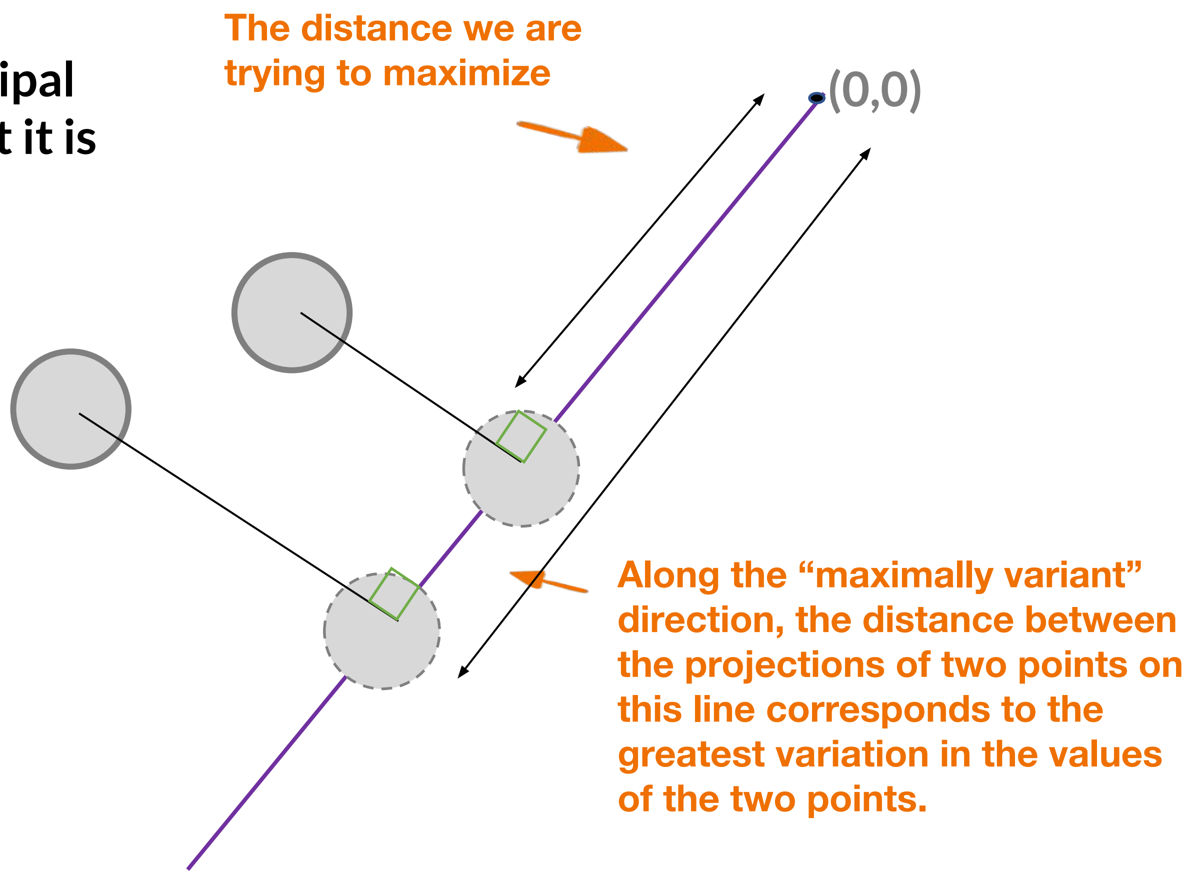
Principal Component Analysis

The projection (A’) of a point A on a particular line p is the point such that the line AA’ is perpendicular to p.

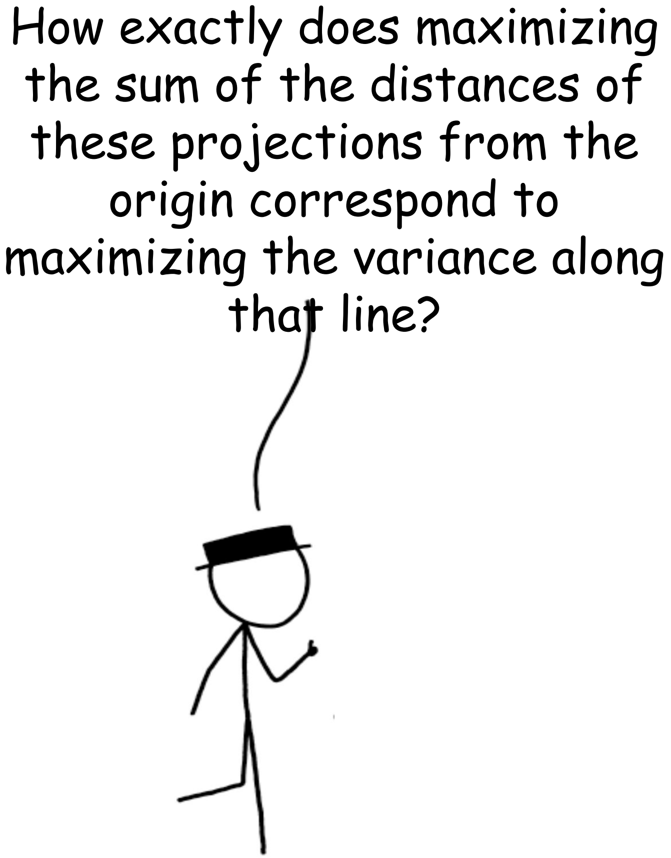


Principal Component Analysis

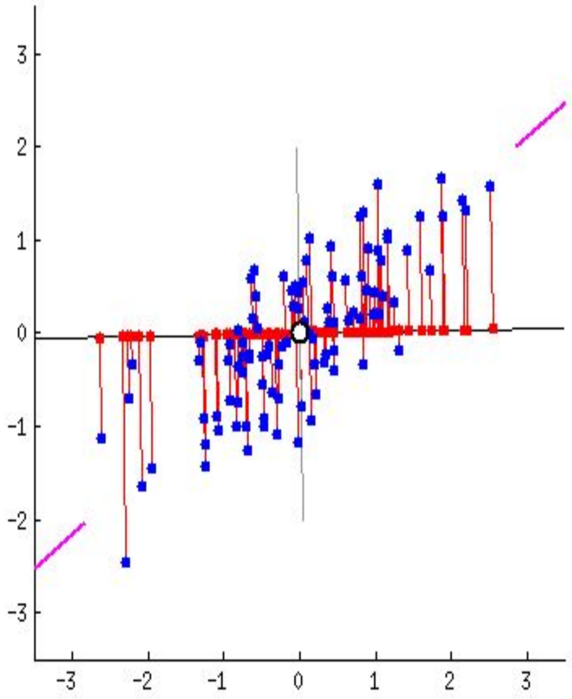
Idea behind this principal component line is that it is an axis along the “maximally variant” direction.

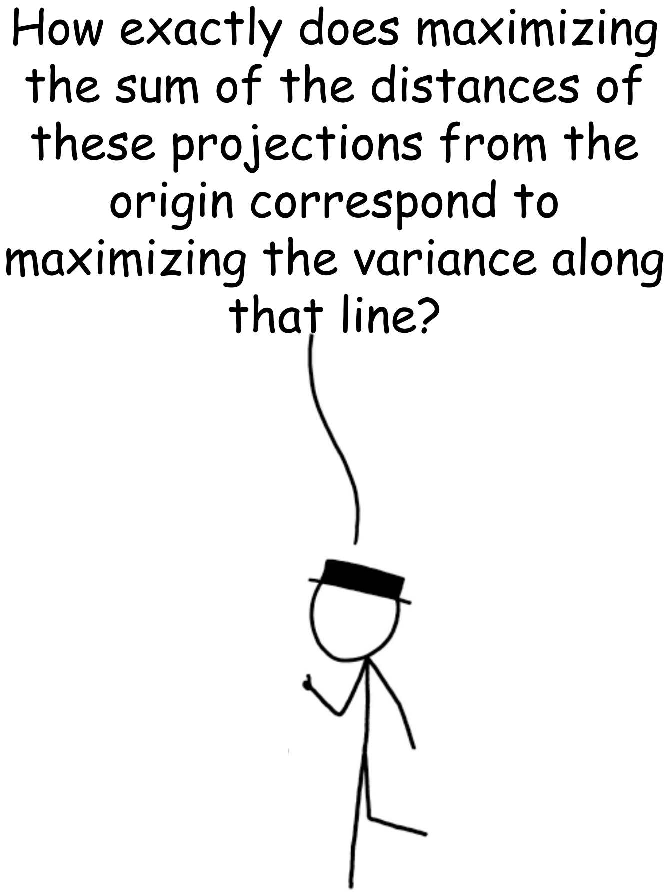


Principal Component Analysis



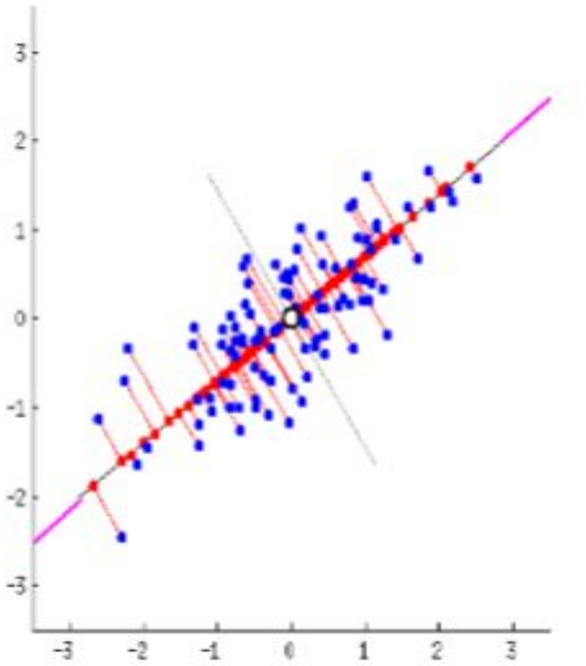
Principal Component Analysis





Built using
https://gist.github.com/anonymous/7d888663c6ec679ea6542871
5b99bfdd

Principal Component Analysis



How exactly does maximizing the sum of the distances of these points from the origin correspond to maximizing the variance along that line?

Principal Component Analysis

Standardization

Covariance Matrix Calculation

Eigenvector Calculation

Form Principal Components and Build Graph

Standardization

Individual	Height (cm)	Weight (kg)	Income ($)	Number of Children
Person A	165	65	60,000	2
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Compare the data of each of the 4 columns. How do they differ numerically?

Standardization

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Range	159-189	63-89	50,000-110,000	1-5
Variance	161.76	135.87	564166666	2.7

Compare the data of each of the 4 columns. How do they differ numerically?

Their range varies drastically. Consequently, their variances are very different.

Standardization

Individual	Height (cm)	Weight (kg)	Income ($)	Number of Children
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Range	159-189	63-89	50k-100k	1-5
Variance	161.76	135.87	564166670	2.7

If this is not addressed, some of the feature columns will **dominate** over the other ones.

This can bias the results and final principal component analysis; making it difficult to view differences between values in one column compared to another.

So final graph may have the differences between the weights of various persons be miniscule.

Standardization

Standardization

Idea: we want to put different variables on the same scale.

This can mean many things from giving them the same mean and standard deviation, to keeping the range consistent, and so on.

Here, we will use a method called **z-scoring.**

$$z = \frac{value - mean}{standard\ deviation}$$

The rescaled distribution will have a mean of 0 and standard deviation of 1

Principal Component Analysis

Standardization

Covariance Matrix Calculation

Eigenvector Calculation

Form Principal Components and Build Graph

Covariance Matrix Calculation

Covariance is really just a measure of how **correlated** two variables/features are.

If your covariance is positive, that means there's a **positive correlation.**

If your covariance is positive, that means there's a **negative correlation.**

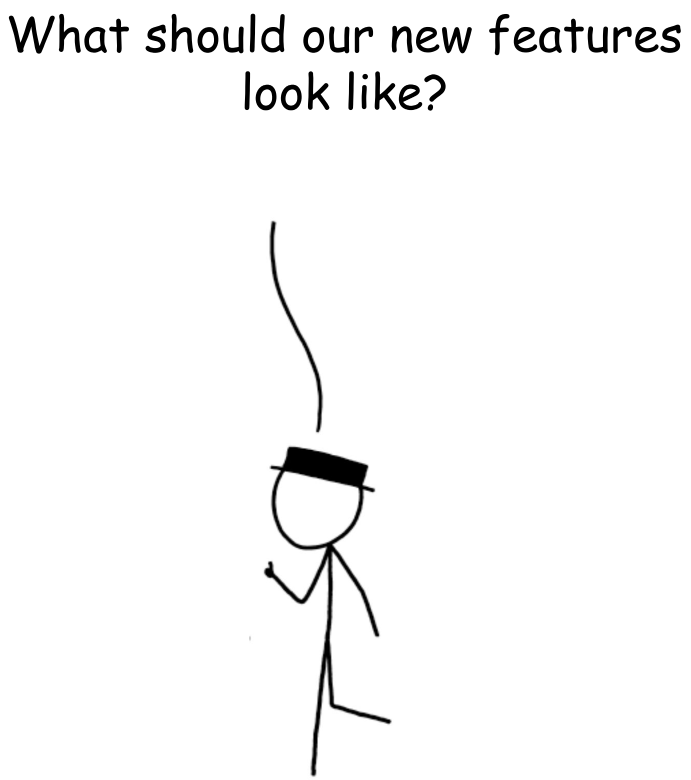
$$Cov(x, y) = \sum \frac{(x_i - \bar{x})(y_i - \bar{y})}{N - 1}$$

Covariance Matrix Calculation

Review: Lecture 7; feature engineering

Make new features with high variance.

Pick new features with low correlation to other features.



Covariance Matrix Calculation

Can measure this correlation using **covariance.** If covariance is **positive,** then features are correlated in the sense they both increase together. If covariance is **negative,** then features are inversely correlated.

$$\begin{bmatrix} Cov(x,x) & Cov(x,y) & Cov(x,z) \\ Cov(y,x) & Cov(y,y) & Cov(y,z) \\ Cov(z,x) & Cov(z,y) & Cov(z,z) \end{bmatrix}$$

$$Cov(x,y) = \sum \frac{(x_i - \bar{x})(y_i - \bar{y})}{N-1}$$

Principal Component Analysis

Standardization

Covariance Matrix Calculation

Eigenvector Calculation

Form Principal Components and Build Graph

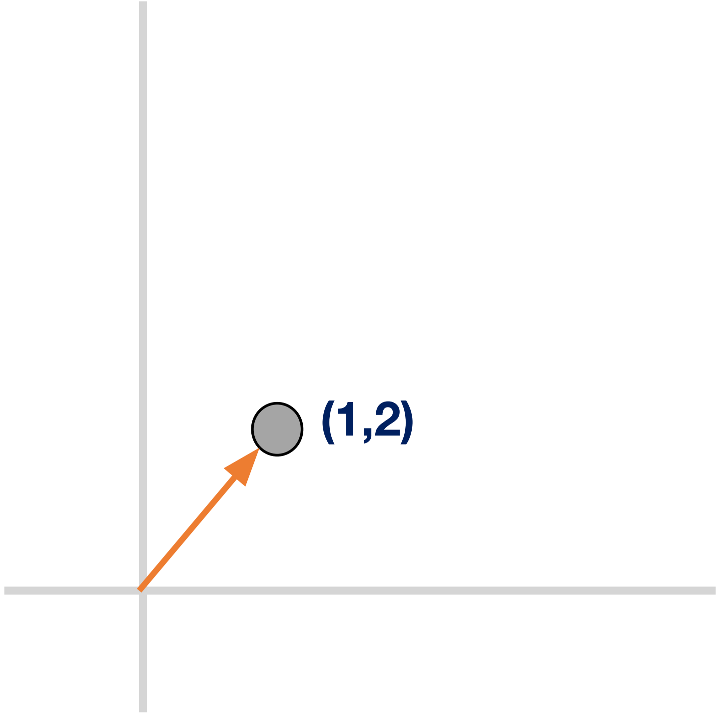
Eigenvector Calculation

We can think of matrices as **transformations** of vectors.

When you multiply a matrix with a vector; two things happen:

1. It **scales** the vector.
2. It **rotates** the vector

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$



Eigenvector Calculation

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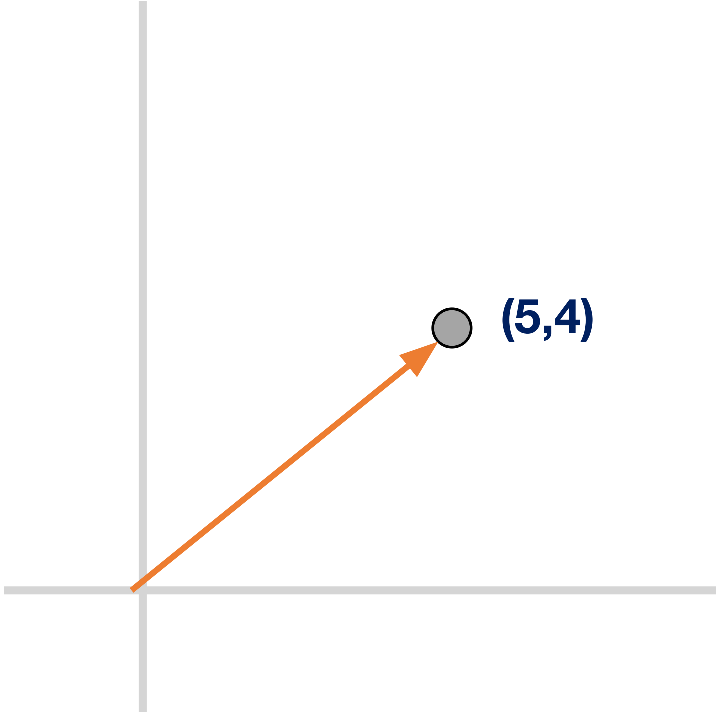
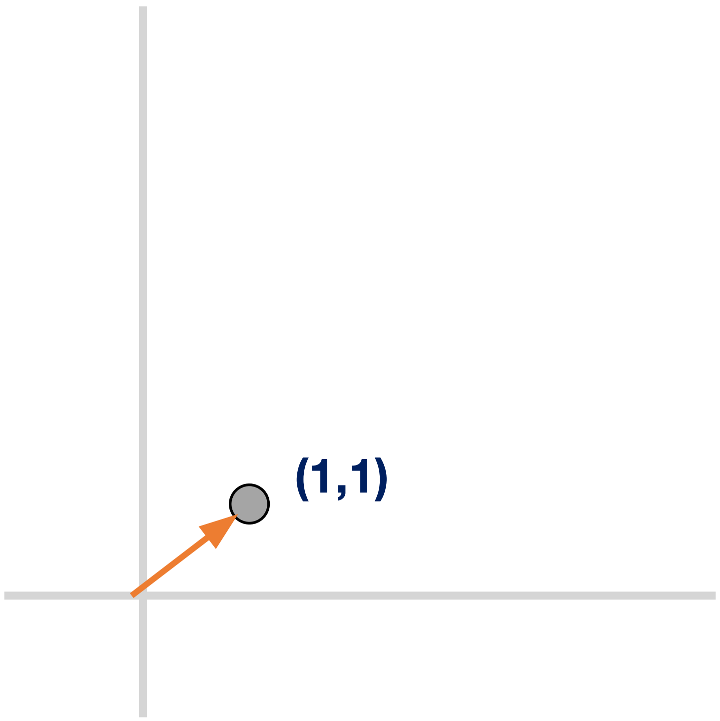


Eigenvector Calculation

Eigenvectors are characteristic vectors specific to a matrix or transformation.

Graphically speaking, when you multiply a matrix with its specific eigenvectors, the eigenvectors **don't get rotated, only scaled.**

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



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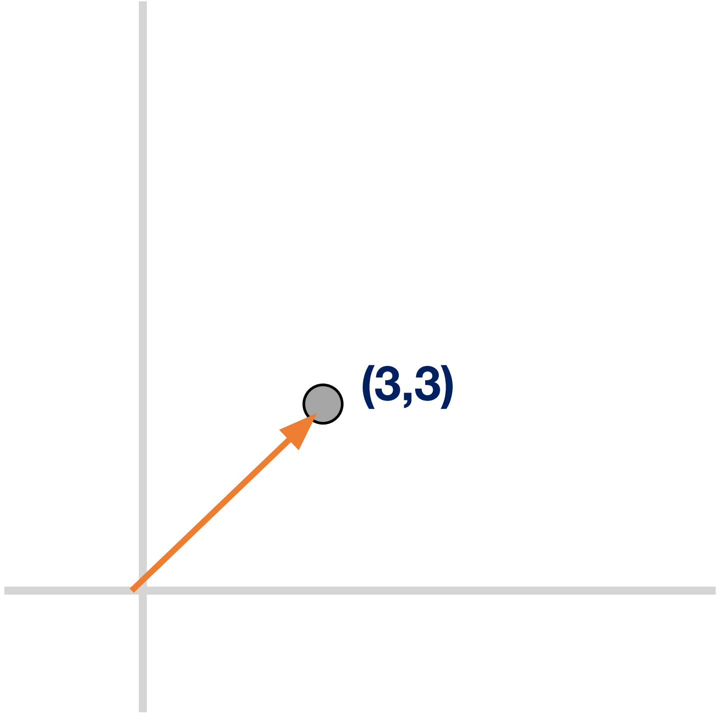
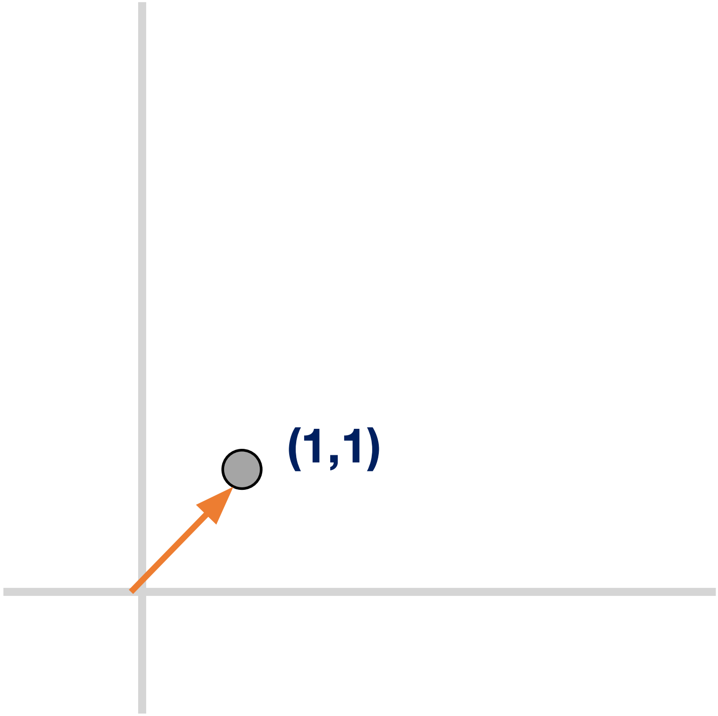
Graphically speaking, when you multiply a matrix with its specific eigenvectors, the eigenvectors **don't get rotated, only scaled.**

The factor by which an eigenvector is scaled is called its eigenvalue

$$\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}3\\ 3\end{bmatrix} = 3\begin{bmatrix}1\\ 1\end{bmatrix}$$



Eigenvector Calculation



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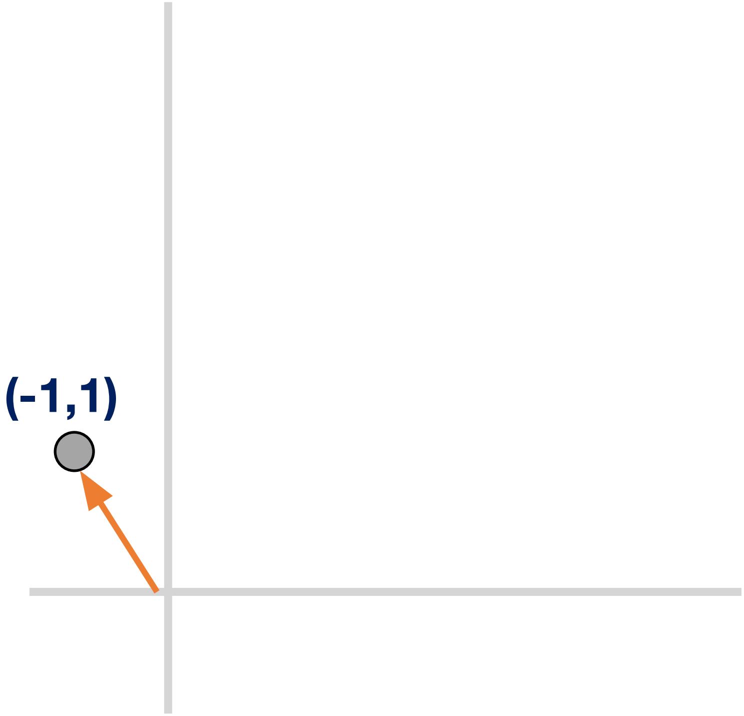
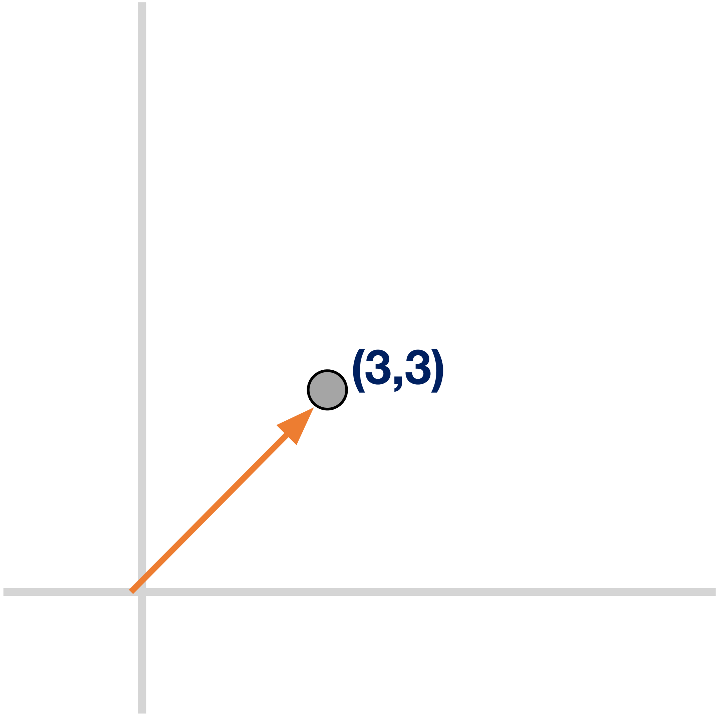


$$\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix} = -1\begin{bmatrix}-1\\ 1\end{bmatrix}$$

Eigenvector Calculation



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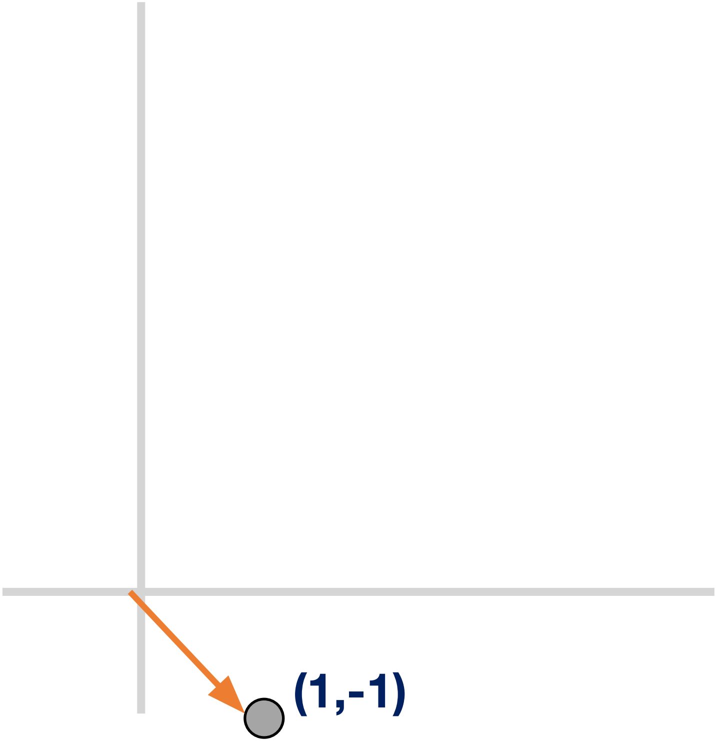

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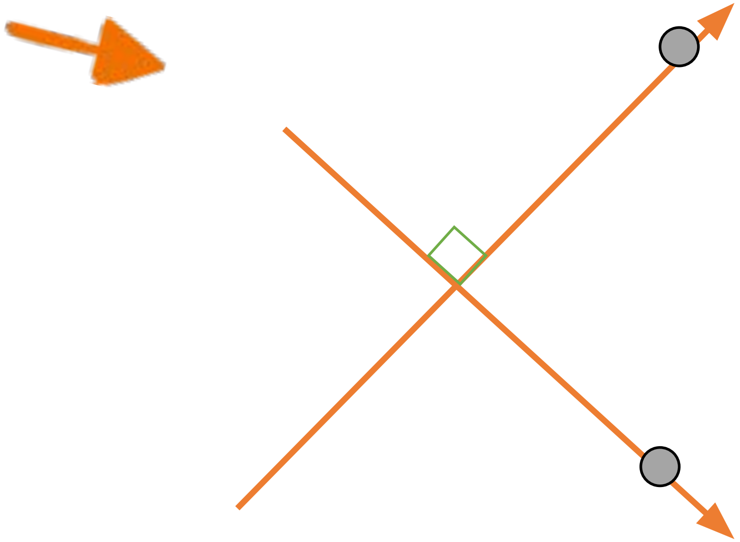
Eigenvector Calculation

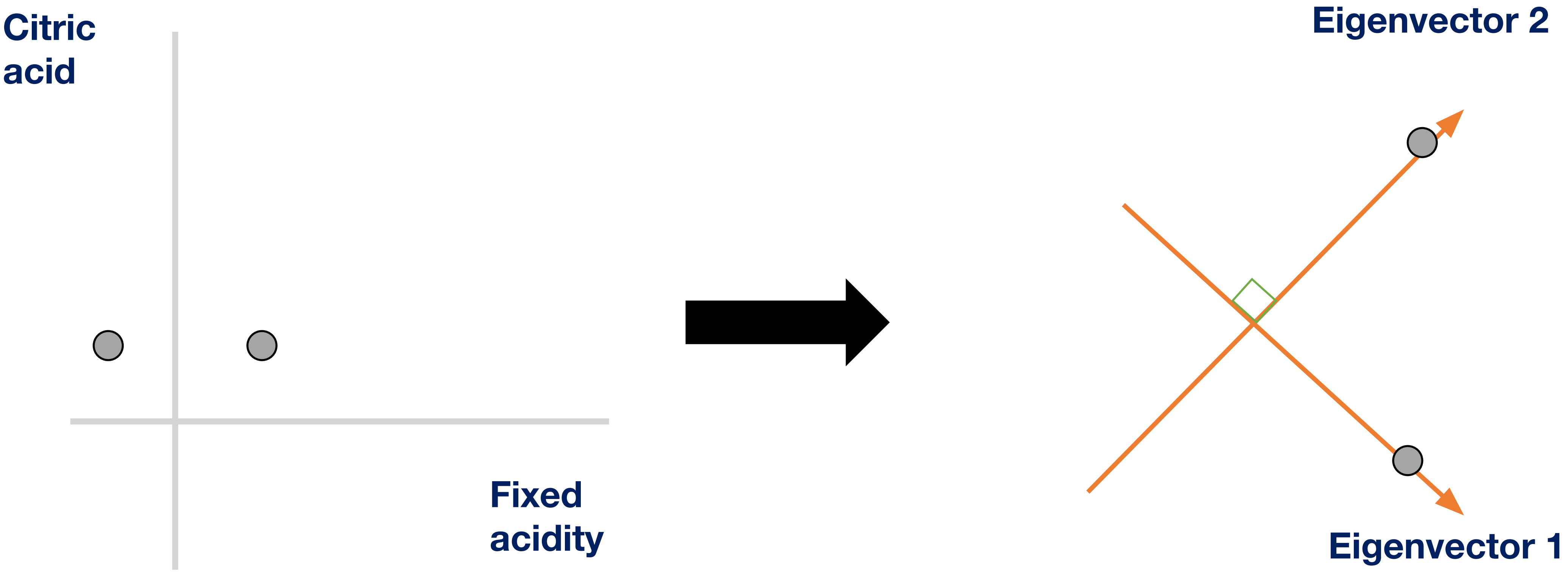
The two eigenvectors are perpendicular to each other!



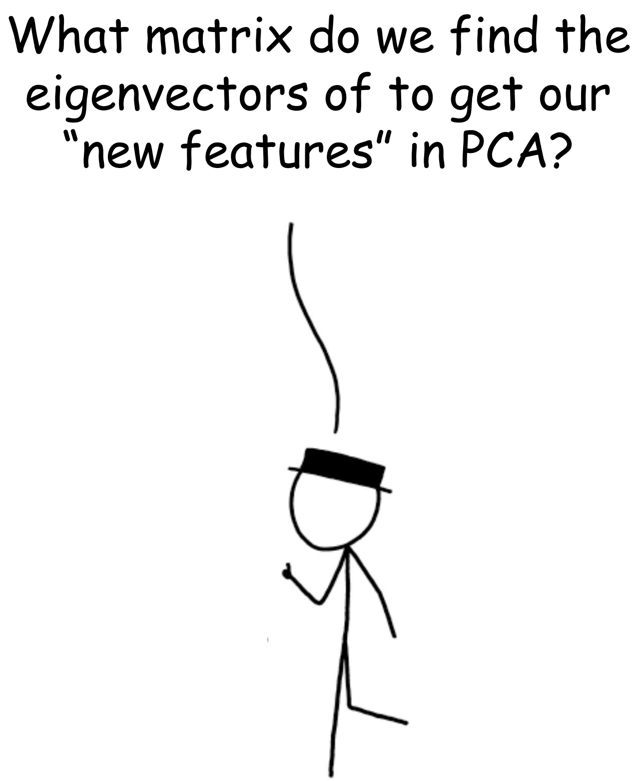
Eigenvectors act as basis vectors!

Every point in 2-D can be expressed as some combination of (1,1) and (-1,1).

Eigenvector Calculation



Eigenvector Calculation



Eigenvector Calculation

By calculating the eigenvectors of the covariance matrix, we can get our **principal components.**

We use the eigenvectors to create a basis for the graph. These basis vectors represent the principal components.

Since these are eigenvectors of the **covariance matrix**, they represent **directions of maximal variance.**

$$A = \begin{bmatrix} Cov(x,x) & Cov(x,y) & Cov(x,z) \\ Cov(y,x) & Cov(y,y) & Cov(y,z) \\ Cov(z,x) & Cov(z,y) & Cov(z,z) \end{bmatrix}$$

$$Av = \lambda v$$

v is the eigenvector and lambda is the eigenvalue

Eigenvector Calculation

$$Av = \lambda v$$

$$Av - \lambda v = 0$$

$$(A - \lambda)v = 0$$

$$|A - \lambda| = 0$$

$$A = \begin{bmatrix} Cov(x,x) & Cov(x,y) & Cov(x,z) \\ Cov(y,x) & Cov(y,y) & Cov(y,z) \\ Cov(z,x) & Cov(z,y) & Cov(z,z) \end{bmatrix}$$

When you find the root of the resulting polynomial, you will find all the possible eigenvalues. For each eigenvalue, plug it into the original equation to find the corresponding eigenvector v.

Principal Component Analysis

Standardization

Covariance Matrix Calculation

Eigenvector Calculation

Form Principal Components and Build Graph

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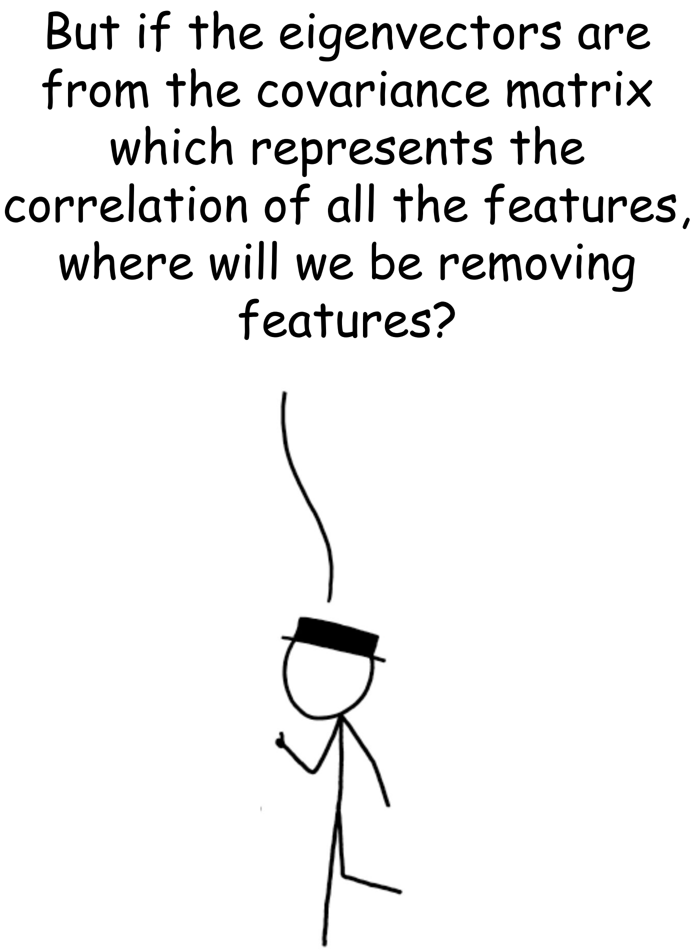
Let the three eigenvalues of the three eigenvectors $v_1, v_{2,}, v_3$ be $\lambda_1, \lambda_2, \lambda_3$ such that $\lambda_1 >= \lambda_2 >= \lambda_3$

Then, the principal components will be $v_1, v_{2,}, v_3$ and the variances they carry are in the ratio of $\lambda_1, \lambda_2, \lambda_3$



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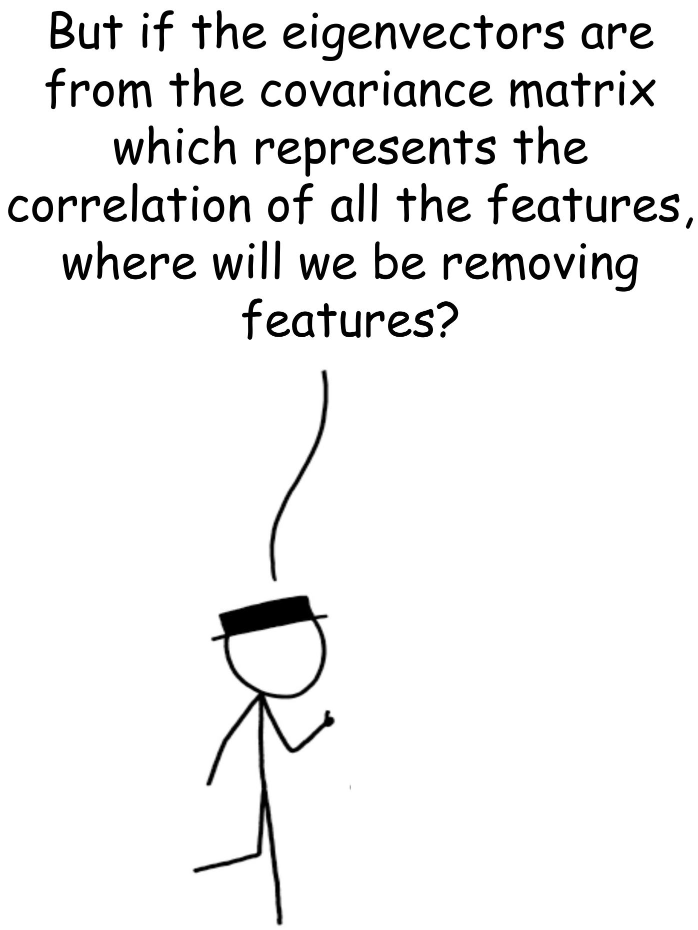
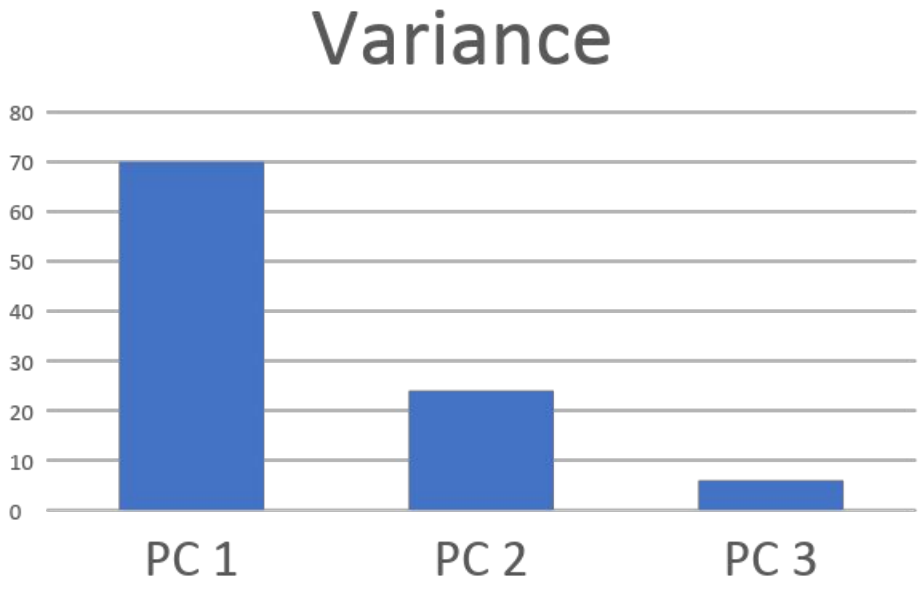


If the percentage of variance of a particular principal component is small enough, discard it. You've now removed a dimension! Form a new matrix which only has the eigenvectors/principal components you've selected.

Let this matrix be called your **Feature Vector.**



Now, it’s time to reorient the original data along these new axes

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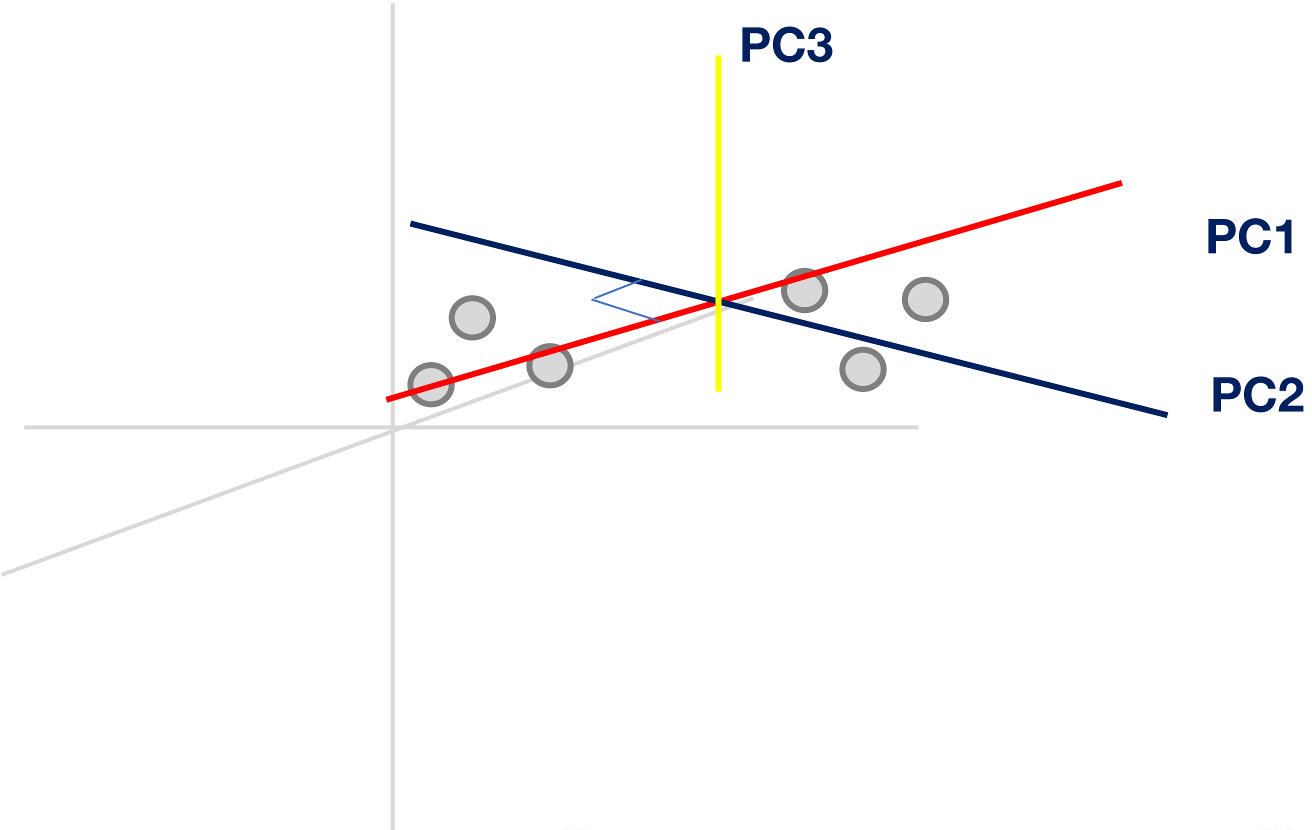

$$FinalDataSet = FeatureVector^T * StandardizedOriginalDataSet^T$$

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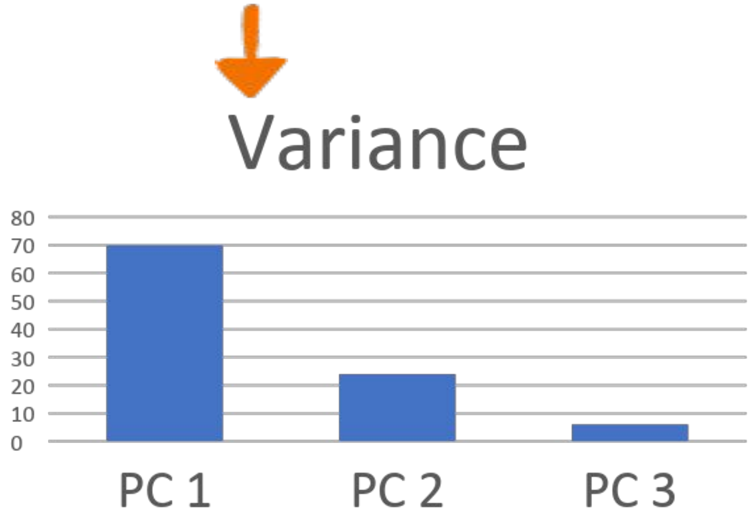
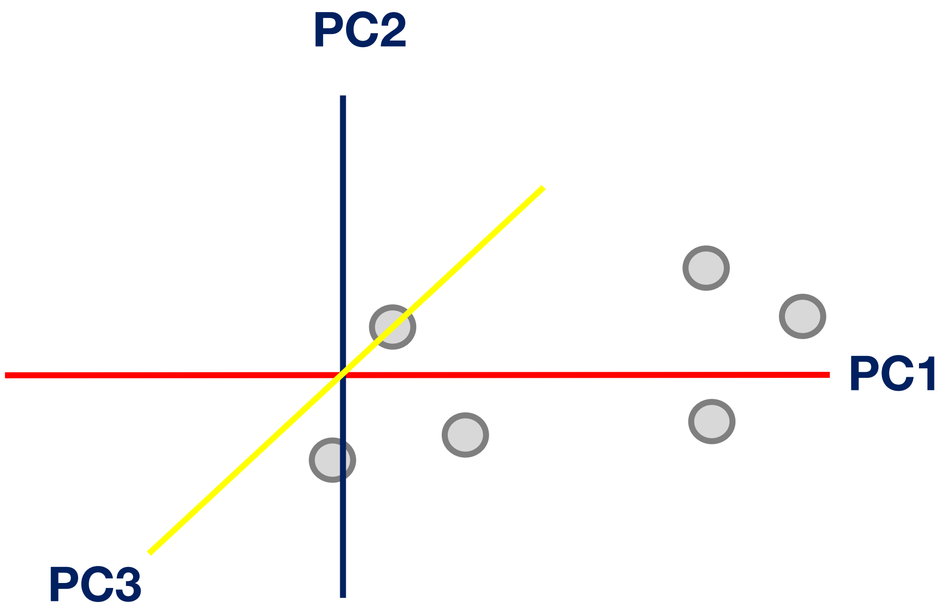
The third dimension was removed as it did not contribute much in terms of variance





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A real-world application of PCA

IQ testing!